PRACTICAL PROBLEMS IN SOIL MECHANICS

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PREFACE

The principal aim of the authors during the production of this volume has been to present in as simple a form as possible the application of soil-mechanics methods and analyses to practical problems such as may occur from day to day in the experience of the practising engineer. The essential features of soil-mechanics studies have been collated and crystallised, omitting mathematics as much as possible and avoiding the analytical approach to the solution of problems. As such, it is anticipated that it will make an appeal both to civil engineers in practice and students studying for civil engineering examinations. It is hoped that this volume will bridge to some extent the gap which exists between the text-book and the use of such theories in actual practice.

The study of earth pressures began in France in 1773, when Coulomb presented his wedge theory of earth pressure and critical height of clay banks, and it is interesting to note that his formulae are used today. Development proceeded, and in 1840 Poncelet completed the wedge theories in a practical manner augmented with the publication of graphs. Collin in 1846, and Airy in 1879, continued investigations with regard to the stability of clay slopes, and for the first time actually measured the shear strength of soils. About this time Darcy (1856) studied the permeability of sands and Rankine (1857) examined the bearing capacity and earth pressure of sands, whilst Boussinesq (1885) investigated the distribution of stress under loaded areas. Important developments took place in 1918, when Professor Krey carried out active research in connection with the construction of the Kiel Canal and evolved stability analyses for foundations, retaining walls and earth slopes. At the same time Bell investigated in Britain the bearing capacity and earth pressure in clays.

A milestone in the study of soils was passed when the science became organised in connection with the Swedish Geotechnical Commission, which was appointed after a number of serious railway accidents, and this Commission published their extensive report in 1922. In America the science was christened "Soil Mechanics" when the First International Conference on Soil Mechanics and Foundation Engineering was held at the Harvard University, Mass., in 1936, when a number of valuable Papers were submitted. Professor Terzaghi reviewed the subject in his James Forrest Lecture at the Institution of Civil Engineers,

London, in 1939, and since that date this Institution has discussed and published many excellent Papers on soil mechanics.

It will be appreciated that the study of soils has now advanced to an organised engineering science. Further research in soil mechanics is now being carried on at the Building Research Station, Watford, and at the Road Research Laboratory, Harmondsworth, under the Department of Scientific and Industrial Research, whilst the Institution of Civil Engineers encourages the presentation of Papers upon the subject.

The Third Edition (1959) contains additional subject-matter on the more recent advances in soil mechanics investigations; such as the provision of sand drains or sand piles for constructing roadways over soils of very weak bearing capacity, the effects of vibrations on soils, the use of piled foundations and the determination of soil properties by nuclear radiation. Further research in these fields are now continuing and reports are printed on such research in the technical press from time to time.

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THE subject-matter of this volume appeared in the technical journal *Civil Engineering* as a series of articles during 1946 and 1947, and in response to many requests the articles have been collected and reprinted. The authors acknowledge their thanks to the proprietors of *Civil Engineering* for permission to reprint these articles, and appreciate the assistance afforded them by the editor of that Journal.

A bibliography of literature on soil mechanics has been included at the end of the volume and the authors have freely referred to these works in the preparation of the text. In each case every endeavour has been made to credit the appropriate authority and to express the authors' own opinions on the subject.

In particular the authors wish to express their thanks to the Director of the Road Research Laboratory of the Department of Scientific and Industrial Research, for permission to include material contained in Chapters V and XIV relating to soil compaction and road surfacings.

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CONTENTS

NOMENCLATURE FOR SOIL MECHANICS

A	Area.
a	Slope of p–e curve. Radius of area of contact of wheel.
B	Breadth.
b	Breadth.
B.S.	British Standard.
C	Coefficient of consolidation.
c	Apparent cohesion.
D	Depth Factor (Taylor).
d	Diameter or distance.
E	Active earth pressure. Young's modulus of elasticity.
e	Voids ratio.
F	Factor of safety. Passive earth resistance.
f_c	Compressive stress per unit area.
G	Factor of safety.
G.W.L.	Ground water level.
H	Horizontal component of a force.
ΔH	Settlement or decrease in thickness.
h	Height or thickness. Head of water.
h_e	Equivalent height.
i	Angle of earth slope. Hydraulic gradient. Infiltration.
J & J′	Consolidation coefficients.
K	Coefficient of permeability.
K′	Coefficient of porosity.
k	Modulus of reaction. Permeability coefficient.
L	Length or length of arc segment.
L.I.	Liquidity index.
L.L.	Liquid limit.
l	Length.
N	Stability number (Taylor). Normal component of force. Number of blows (Liquid limit test).
N_A N_B N_C	Factors for settlement.
n	Porosity. Coefficient of viscosity of a liquid.
P	Concentrated load or force. Total water pumped.
P.I.	Plasticity index.

P.L.	Plastic limit.
p	Unit load or pressure.
Q_v	Vertical load.
Q_h	Horizontal or sliding force.
q	Bearing pressure. Total discharge.
q_u	Ultimate bearing capacity.
R	Radius.
r	Radius of influence.
S	Slope of ground water table.
S.L.	Shrinkage limit.
s	Shear resistance per unit area. Degree of saturation.
s′	Skin friction between soil and foundation.
s_m	Specific mass gravity (apparent specific gravity).
T	Tangential component of a force. Tie bar pull. Time factor.
t	Time interval.
u	Percentage consolidation.
V	Volume. Coefficient of constant consolidation.
v	Velocity. Vertical component of a force.
W	Weight. Moisture content.
w	Weight per unit area.
Z	Vertical pressure at depth D.
α	Angle of repose. Angle for Fellenius' centre.
β	Angle for Fellenius' centre.
γ	Unit weight of soil.
γ_w	Unit weight of water.
δ	Angle of friction between soil and wall.
μ	Poisson's ratio.
σ	Normal pressure. Horizontal pressure.
ρ	Specific gravity of soil grains.
ρ_w	Specific gravity of liquid.
ϕ	Angle of internal friction.
ϕ_w	Weighted angle of internal friction.
Σ	Sum of or total.
Ƶ	Depth of tension crack.

CHAPTER I

SOIL CHARACTERISTICS AND PROPERTIES

It has been the experience of the authors, when discussing soil mechanics with engineers taking an initial interest in the subject, to find that the value of soil classification and experimental work to ascertain soil properties is not always adequately appreciated at the outset, although at a later stage, when the study of the subject has progressed, the importance of this section of soil mechanics becomes fully realised.

It is essential to become familiar with the results of soil tests and to understand their significance, and when experience has been gained one is able to form definite opinions from such data relating to moisture content, plastic limit, liquid limit, etc., whether a certain soil is suitable for tipping in an embankment of a specified height, whether the load of a structure is capable of being supported without undue or uneven settlement, whether ground water lowering or artificial cementations for deep excavations is possible, and other similar foundation problems.

The first two chapters will be allocated to examples involving soil classification and soil properties which will provide a basis for the practical engineering problems contained in the later chapters.

Soil Classification

Soil consists of a mixture of mineral particles and water, and includes a wide range of materials from shingle to plastic clay. In the study of soil mechanics it is most important to be able to classify the soils into certain well-defined types dependent on the size, shape and nature of the particles, but it must be understood also that the properties of a soil depend largely on its moisture content.

Classification tests are of two types :—

(1) Mechanical analysis, by means of sieving or sedimentation to determine the size-distribution of the soil particles.

(2) Index tests, for soils passing a 36-mesh B.S. sieve, by means of which the type is deduced from the moisture content at standard consistencies.

The generally accepted standards are given in Tables 1 and 2 together with the chart in Fig. 1.

TABLE 1.

Soil Classification.

Soil description.	Diameter of particle or sieve size.	Notes.
Boulders	Greater than $2\frac{1}{2}$ in.	—
Gravel	From $2\frac{1}{2}$ in. to $\frac{1}{8}$ in. (or No. 7 B.S. sieve).	—
Coarse sand	From No. 7 to No. 25 B.S. sieve.	—
Medium sand	From No. 25 to No. 72 ,, ,,	—
Fine sand	From No. 72 to No. 200 ,, ,,	—
Silt	Particles pass No. 200 B.S. sieve but are larger than 0·002 mm.	Silts have practically no permeability.
Clay	Particles are smaller than 0·002 mm.	Clays exhibit plasticity and cohesion.

TABLE 2.

Classification of Soils.

Soil description.	Percentage of soil separates present.		
	Sand.	Silt.	Clay.
Sand	80–100	0– 20	0– 20
Sandy loam	50– 80	0– 50	0– 20
Sandy clay loam	50– 80	0– 30	20– 30
Sandy clay	50– 70	0– 20	30– 50
Clay	0– 50	0– 50	30–100
Silty clay	0– 20	50– 70	30– 50
Silty clay loam	0– 30	50– 80	20– 30
Silty loam	0– 50	50– 80	0– 20
Silt	0– 20	80–100	0– 20
Clay loam	20– 50	20– 50	20– 30
Loam	30– 50	30– 50	0– 20

Sedimentation.

The grains of a soil settle in a liquid with a velocity which may be calculated by Stoke's law, which states that the rate at which a small sphere sinks in a liquid is directly proportional to the square of the diameter of the sphere. This law applies only when considering grain diameters between 0·2 mm. and 0·0002 mm. Grains larger than 0·2 mm. diameter settle with varying velocity and particles less than 0·0002 mm. diameter are in colloidal suspension.

Velocity of settling in cms. per sec.

$$v = \frac{2(\rho - \rho_w)}{9n}\left(\frac{d}{2}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where ρ denotes specific gravity of the soil grains,
ρ_w ,, ,, ,, ,, liquid,
n ,, coefficient of viscosity for the liquid,
(= 0·000103 kg. sec. per sq. m. for water at 20° C.)
d ,, diameter of the soil grains.

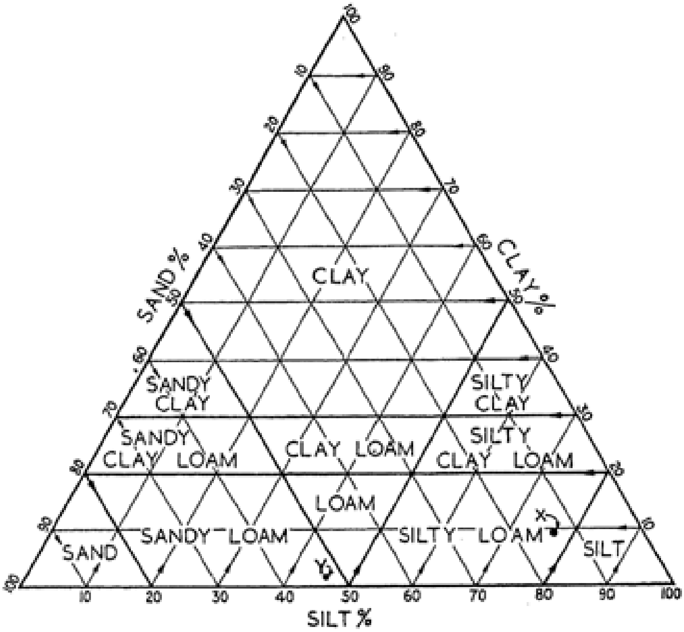


Fig. 1.—Soil Classification Chart.

The simplified formula for spherical particles descending in still water is as follows :—

$$v = 8800\ d^2 \text{ or } d = \frac{\sqrt{v}}{94} \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Problem 1. How long would it take for a particle of soil 0·01 mm. in diameter to settle from the surface to the bottom of a pond 10 ft. deep, if the specific gravity of the water is 1·00 and

of the soil is 2·55, and the coefficient of viscosity of water is 0·1025 g.sec. per sq. m.? (Check by simplified formula.)

$$\rho = 2{\cdot}55 \qquad \rho_w = 1{\cdot}00 \qquad d = 0{\cdot}01 \text{ mm.} = 0{\cdot}001 \text{ cm.}$$
$$n = 0{\cdot}1025 \text{ g. sec./sq. m.} = 0{\cdot}00001025 \text{ g. sec./sq. cm.}$$
$$v = \frac{2(2{\cdot}55 - 1)}{9 \times 0{\cdot}00001025} \times \left(\frac{0{\cdot}001}{2}\right)^2 = 0{\cdot}0084 \text{ cm./sec.}$$
$$(\text{Check :—} v = 8800 \times 0{\cdot}001^2 = 0{\cdot}0088 \text{ cm./sec.})$$

Length of time for particle to settle

$$= \frac{10 \times 12 \times 2{\cdot}54}{0{\cdot}0084} = 36{,}285 \text{ sec.} = 10{\cdot}08 \text{ hrs.}$$

Problem 2. The results of a sedimentation test on a sample passing a No. 200 B.S. sieve are set out below. Neglecting the effects of temperature variations, ascertain the grain size distribution.

Reading.	Time.	Depth at which sample is taken by a 20 c.c. pipette.	Amount of solid particles in the sample of 20 c.c. taken by pipette obtained on drying.
i	Zero	At any depth	20 grms.
ii	45 sec.	10 cms.	11·5 ,,
iii	4 mins. 46 secs.	10 ,,	7·0 ,,
iv	9 mins. 30 secs.	5 ,,	3·5 ,,
v	4 hrs.	5 ,,	0·2 ,,

Velocity of settling, $v = \frac{\text{Depth}}{\text{Time}}$

For reading i, commencement of test.

,, ,, ii, $v = 10 \div 45 = 0{\cdot}222$ cm. per sec.

,, ,, iii, $v = 10 \div 286 = 0{\cdot}0384$,,

,, ,, iv, $v = 5 \div 570 = 0{\cdot}0088$,,

,, ,, v, $v = 5 \div 14400 = 0{\cdot}00034$,,

From equation (2) $d = \frac{\sqrt{v}}{94}$

$\therefore$ particle size, d for reading—

ii $= \sqrt{0{\cdot}222} \div 94 = 0{\cdot}0047$ cm. $= 0{\cdot}047$ mm.

iii $= \sqrt{0{\cdot}0384} \div 94 = 0{\cdot}00208$,, $= 0{\cdot}0208$,,

iv $= \sqrt{0{\cdot}0088} \div 94 = 0{\cdot}001$,, $= 0{\cdot}01$,,

v $= \sqrt{0{\cdot}00034} \div 94 = 0{\cdot}0002$,, $= 0{\cdot}002$,,

Thence the percentage of particles "finer than":—

No. 200 B.S. sieve = 0·076 mm.	0·047 mm.	0·0208 mm.	0·01 mm.	0·002 mm.
100%	$\frac{11\cdot5}{20} = 57\cdot5\%$	$\frac{7}{20} = 35\%$	$\frac{3\cdot5}{20} = 17\cdot5\%$	$\frac{0\cdot2}{20} = 1\%$

Problem 3. The results of the mechanical analysis of two soil samples, which will be termed X and Y respectively, were as follows:—

	X.	Y.
Retained on sieve No. 20	3 grs.	6 grs.
,, ,, No. 60	6 ,,	20 ,,
,, ,, No. 140	9 ,,	32 ,,

The sedimentation tests showed that the samples contained:—

	X.	Y.
d = 0·05 mm. to 0·005 mm.	67 grs.	45 grs.
d = 0·005 mm. to 0·002 mm.	30 ,,	5 ,,
d = less than 0·002 mm.	12 ,,	2 ,,

Total weight of oven-dried sample: X = 71 grs. Y = 73 grs. Calculate percentage particle sizes and plot grain-size curves. Find the classification of these two soils according to the chart in Fig. 1.

Particle size.	Sample X.			Sample Y.		
	Weight.	% retained.	% finer than.	Weight.	% retained.	% finer than.
B.S. sieve No. 20	3 grs.	2·36	97·64	6 grs.	5·45	94·55
,, No. 60	6 ,,	4·72	92·92	20 ,,	18·18	76·37
,, No. 140	9 ,,	7·09	85·83	32 ,,	29·09	47·28
0·05 to 0·005 mm.	67 ,,	52·76	33·07	45 ,,	40·91	6·37
0·005 to 0·002 mm.	30 ,,	23·63	9·44	5 ,,	4·55	1·82
Less than 0·002 mm.	12 ,,	9·44	Clay	2 ,,	1·82	Clay
Total	127 grs.	100%	—	110 grs.	100%	—

Grain-size curves for X and Y are plotted on Fig. 2.
Sample X according to chart in Fig. 1 is a "silty loam."
Sample Y according to chart in Fig. 1 is a "sandy loam."

Problem 4. In Fig. 2 grain-size curves are plotted for three samples of soils A, B and C; from inspection of these curves give the effective diameter and the coefficient of uniformity of these three soil types.

(*Note.*—The effective diameter is the particle size for 10 per cent.

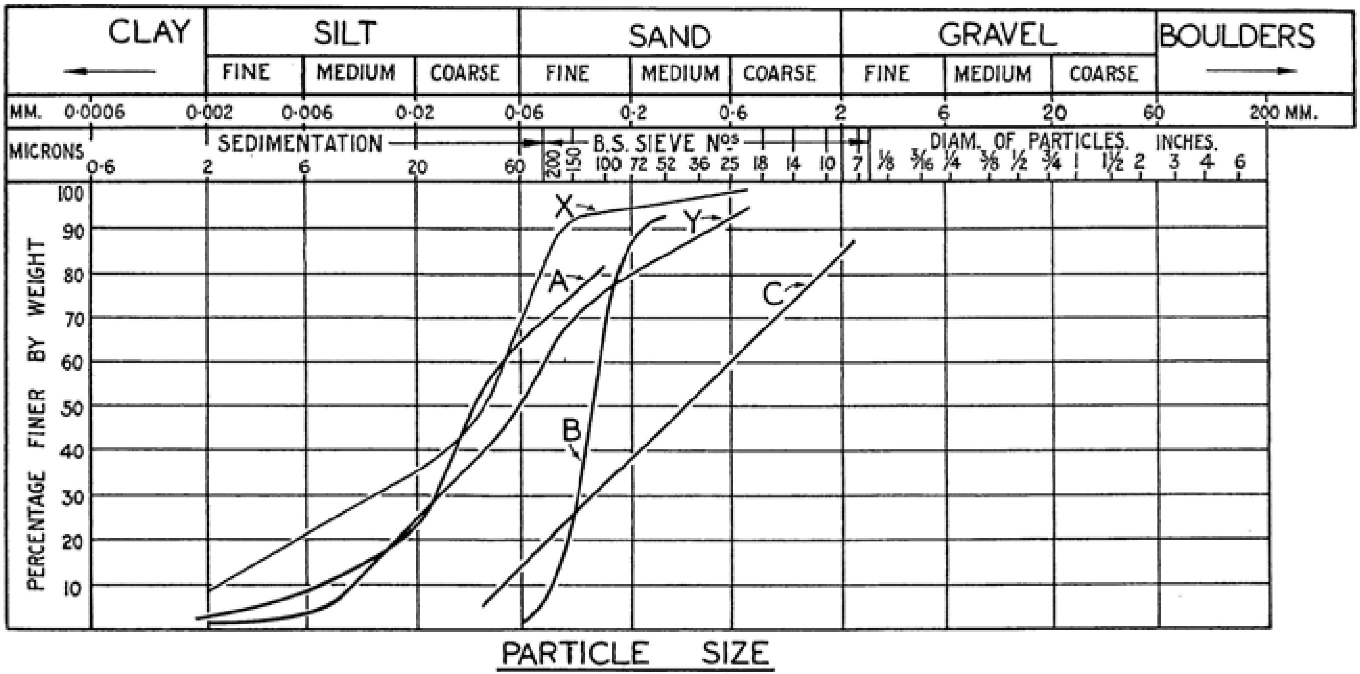


FIG. 2.—Grain-size Chart.

finer by weight. The coefficient of uniformity is the ratio of the particle size for 60 per cent. finer by weight to the effective diameter.)

From inspection of Fig. 2 the following table can be drawn up :—

Sample.	Effective diam. = diam. for 10% finer.	Diameter for 60% finer.	Coefficient of uniformity.
A	0·007 mm.	0·055 mm.	$\frac{0·055}{0·007} = 7·86$
B	0·096 mm.	0·146 mm.	$\frac{0·146}{0·096} = 1·52$
C	0·056 mm.	0·63 mm.	$\frac{0·63}{0·056} = 11·25$

Soil Properties. Voids Ratio, Porosity, Moisture Content and Density.

Voids Ratio and Porosity.

A soil is made up of soil particles with the voids between the particles filled with either moisture or air, or both.

The voids ratio, $e = \frac{\text{Volume of voids}}{\text{Volume of solids}}$

and the porosity, $n = \frac{\text{Voids}}{\text{Total volume}}$

$$\therefore e = \frac{n}{1-n} \text{ and } n = \frac{e}{1+e} \quad . \quad . \quad . \quad (3)$$

Moisture Content.

The natural moisture content of a soil is determined by weighing a sample before and after drying at 105° C. The loss in weight is expressed as a percentage of the dry weight. When a soil is saturated, the moisture content,

$$W = \frac{e}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where ρ denotes the absolute density (or specific gravity of the grains).

Densities.

The following formulæ in terms of voids ratio or porosity will enable most problems in densities to be solved.

The volume of solids $= 1 - n = \frac{1}{1+e}$

The weight of solids $= \rho(1 - n)\gamma_w = \frac{\rho}{1 + e}\gamma_w$

where γ_w denotes the unit weight of water ($= 62{\cdot}5$ lb. per cu. ft.).

(*a*) If the voids are saturated, then

$$\text{the weight of water} = n\gamma_w = \frac{e}{1 + e}\gamma_w \quad . \quad . \quad . \quad (5)$$

$$\text{the dry density} = \rho(1 - n)\gamma_w = \frac{\rho}{1 + e}\gamma_w \quad . \quad (6)$$

$$\left.\begin{array}{l}\text{the bulk density or}\\ \text{saturated density}\end{array}\right\} = [\rho(1 - n) + n]\gamma_w = \frac{\rho + e}{1 + e}\gamma_w \quad . \quad . \quad . \quad (7)$$

$$\left.\begin{array}{l}\text{and the submerged}\\ \text{density}\end{array}\right\} = (\rho - 1)\,(1 - n)\gamma_w = \frac{\rho - 1}{1 + e}\gamma_w \quad . \quad . \quad . \quad (8)$$

(*b*) If the voids are partly saturated, let s denote the degree of saturation, then

$$\text{the weight of water} = \frac{s.e}{1 + e}\gamma_w \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\text{the moisture content, } W = s\frac{e}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\left.\begin{array}{l}\text{and the bulk density or}\\ \text{saturated density}\end{array}\right\} = \frac{\rho + s.e}{1 + e}\gamma_w = \frac{\rho(1 + W)}{1 + e}\gamma_w \quad . \quad . \quad . \quad . \quad (11)$$

Problem 5. The porosity n of a coarse sand is 27 per cent. Determine its voids ratio e.

$$e = \frac{n}{1 - n} = \frac{0{\cdot}27}{1 - 0{\cdot}27} = 0{\cdot}37 = 37\%$$

Problem 6. Determine the maximum value of the voids ratio e for a sand composed of grains assumed to be perfectly spherical.

$$e = \frac{\text{Volume of voids}}{\text{Volume of solids}}$$

$$= \frac{(2r)^3 - \left(\frac{1}{8} \times \frac{\pi d^3}{6}\right)8}{\left(\frac{1}{8} \times \frac{\pi d^3}{6}\right)8} = \frac{d^3 - \frac{\pi d^3}{6}}{\frac{\pi d^3}{6}} = 0{\cdot}91 = 91\%$$

Problem 7. The specific mass gravity (or bulk density) of a soil sample is 1·67. The absolute density (or specific gravity of the grains) is 2·74. Determine the voids ratio e under the assumption that the soil is perfectly dry.

$$e = \frac{\text{Volume of voids}}{\text{Volume of solids}} = \frac{\text{Total volume} - \text{Volume of solids}}{\text{Volume of solids}}$$

$$= \frac{2{\cdot}74}{1{\cdot}67} - 1 = 0{\cdot}64 = 64\%$$

Alternatively, as the soil is perfectly dry, the bulk density $= \frac{\rho}{1 + e}$

$$\therefore\ 1{\cdot}67 = \frac{2{\cdot}74}{1 + e} \qquad \therefore\ e = \frac{2{\cdot}74}{1{\cdot}67} - 1 = 0{\cdot}64 = 64\%$$

Problem 8. A clay sample was found to weigh 42·353 grs. before drying and 33·765 grs. after drying in an electric oven at 105° C. What was the moisture content of the sample?

Moisture content,

$$W = \frac{\text{Weight of water}}{\text{Dry weight of sample}} = \frac{42{\cdot}353 - 33{\cdot}765}{33{\cdot}765}$$
$$= 0{\cdot}254 = 25{\cdot}4\%$$

Problem 9. A sand sample has a porosity of 35 per cent. and an absolute density of 2·7. What is the dry density, bulk density and submerged density of the sample?

$$\text{Dry density} = \rho(1 - n)\gamma_w = 2{\cdot}7\ (1 - 0{\cdot}35)\ 62{\cdot}5 = 109{\cdot}7 \text{ lb./cu. ft.}$$
$$\text{Bulk density} = [\rho(1 - n) + n]\gamma_w = [2{\cdot}7(1 - 0{\cdot}35) + 0{\cdot}35]62{\cdot}5 = 131{\cdot}56 \text{ lb./cu. ft.}$$
$$\text{Submerged density} = (\rho - 1)\ (1 - n)\gamma_w = (2{\cdot}7 - 1)\ (1 - 0{\cdot}35)\ 62{\cdot}5 = 69{\cdot}06 \text{ lb./cu. ft.}$$

Problem 10. A clay sample has a moisture content of 45 per cent., voids ratio of 1·22 and absolute density of 2·7. Assuming saturation, find the dry density, bulk density and submerged density of the clay.

$$\text{Dry density} = \frac{\rho}{1 + e}\gamma_w = \frac{2{\cdot}7}{1 + 1{\cdot}22}\,62{\cdot}5 = 76 \text{ lb./cu. ft.}$$

$$\text{Bulk density} = \frac{\rho + e}{1 + e}\gamma_w = \frac{2{\cdot}7 + 1{\cdot}22}{1 + 1{\cdot}22}\,62{\cdot}5 = 110 \text{ lb./cu. ft.}$$

$$\text{Submerged density} = \frac{\rho - 1}{1 + e}\gamma_w = \frac{2{\cdot}7 - 1}{1 + 1{\cdot}22} 62{\cdot}5 = 48 \text{ lb./cu. ft.}$$

Problem 11. A clay fill has a bulk density of 120 lb./cu. ft. If the moisture content is 27·5 per cent. and the absolute density is 2·7, find the degree of saturation.

$$\text{From equation (10)} \quad e = \frac{W \times \rho}{s} = \frac{0{\cdot}275 \times 2{\cdot}7}{s} = \frac{0{\cdot}7425}{s}$$

$$\text{From equation (11) Bulk density} = \frac{\rho + s \times e}{1 + e}\gamma_w$$

$$120 = \frac{2{\cdot}7 + s \times e}{1 + e} 62{\cdot}5$$

$$= \frac{2{\cdot}7 + s\left(\frac{0{\cdot}7425}{s}\right)}{1 + \frac{0{\cdot}7425}{s}} 62{\cdot}5$$

Degree of saturation, s = 0·94 = 94 per cent.

Problem 12. A sample of moist (but not fully saturated) soil has a volume of 60 c.c. and weighs 92·5 grs. After complete drying out in an oven its weight is 74·3 grs. The absolute density is 2·62. Calculate the degree of saturation.

Volume of soil + Volume of water + Volume of air = 60 c.c.

Weight of soil + Weight of water = 92·5 grs.

Weight of soil = 74·3 grs.

$$\text{Absolute density} = \frac{\text{Weight of soil}}{\text{Volume of soil}} = 2{\cdot}62$$

Weight of water = 92·5 − 74·3 = 18·2 grs.

In metric units, Weight of water in grs. = Volume of water in c.c. = 18·2 c.c.

$$\therefore \text{ Volume of soil} = \frac{\text{Weight of soil}}{2{\cdot}62} = \frac{74{\cdot}3}{2{\cdot}62} = 28{\cdot}36 \text{ c.c.}$$

∴ Volume of air + Volume of water = 60 − 28·36 = 31·64 c.c.

$$\therefore \text{ Degree of saturation} = \frac{\text{Volume of water in pores}}{\text{Total volume of pores}}$$

$$= \frac{18{\cdot}2}{31{\cdot}64} = 0{\cdot}575 = 57{\cdot}5\%$$

Pycnometer Bottle.

To determine the specific gravity of soil particles, a pycnometer bottle is used. This consists of a flask which has a volume of

500 c.c. at a certain temperature, usually 20° C., and this volume is marked on the neck of the bottle. A soil sample of 25 to 50 grs. is placed in the bottle, which is then filled up with distilled water. The liquid is boiled to expel the air adhering to the soil particles, and when cool the bottle is filled up to the mark on the neck and weighed.

The specific gravity of the soil grains,

$$= \frac{\text{Weight of dry soil in bottle}}{\left\{\begin{matrix}\text{Wgt. of dry} \\ \text{soil in bottle}\end{matrix}\right\} + \left\{\begin{matrix}\text{Wgt. of bottle} \\ + \text{Water}\end{matrix}\right\} - \left\{\begin{matrix}\text{Wgt. of bottle} \\ + \text{sample} + \text{water}\end{matrix}\right\}}$$

Problem 13. The following data are obtained from tests on a soil sample in its natural state: volume 887 c.c., weight 1,541 grs.; in a loose, dry state: volume 1,052 c.c., weight 1,311 grs.; and in a dense, dry state: volume 632 c.c., weight 1,311 grs. Weight of pycnometer bottle and water 663·23 grs. Weight of pycnometer bottle 40 grs., of soil and water for same volume 687·29 grs.

Find the specific gravity, voids ratio, relative humidity and relative density of the soil sample.

(*a*) Specific gravity

$$= \frac{\text{Weight of dry soil sample in bottle}}{\left\{\begin{matrix}\text{Wgt. of dry soil} \\ \text{sample in bottle}\end{matrix}\right\} + \left\{\begin{matrix}\text{Wgt. of bottle} \\ \text{and water}\end{matrix}\right\} - \left\{\begin{matrix}\text{Wgt. of bottle} + \\ \text{sample} + \text{water}\end{matrix}\right\}}$$

$$= \frac{40}{40 + 663\cdot23 - 687\cdot29} = \frac{40}{15\cdot94} = 2\cdot51$$

(*b*) Volume of solids $= \frac{\text{Wgt. of solids}}{\text{Specific gravity}} = \frac{1311}{2\cdot51} = 522\cdot31$ c.c.

$$\text{Voids ratio, } e = \frac{\text{Volume of voids}}{\text{Volume of solids}}$$

$$= \frac{\text{Vol. of air} + \text{Vol. of water} + \text{Vol. of solids}}{\text{Volume of solids}} - 1$$

$$= \frac{887}{522\cdot31} - 1 = 1\cdot7 - 1 = 0\cdot7$$

(*c*) Relative humidity

$$= \frac{\text{Weight of water per unit volume of solid material}}{\text{Voids ratio}}$$

$$= \frac{\dfrac{230 \times 2\cdot51}{1311}}{0\cdot7} = \frac{230 \times 2\cdot51}{1311 \times 0\cdot7} = \frac{577\cdot3}{917\cdot7} = 0\cdot63$$

(*d*) Relative density

$$= \frac{\text{Max. voids ratio loose} - \text{Natural voids ratio}}{\text{Max. voids ratio loose} - \text{Min. voids ratio dense}}$$

Maximum—

$$e = \frac{\text{Vol. of air} + \text{Vol. of solids}}{\text{Vol. of solids}} - 1$$

$$= \frac{1052}{\text{Wgt. of solids} \div \text{Specific gravity}} - 1$$

$$= \frac{1052 \times 2{\cdot}51}{1311} - 1 = 1{\cdot}01$$

Minimum—

$$e = \frac{6{\cdot}32 \times 2{\cdot}51}{1311} - 1 = 0{\cdot}21$$

Relative density—

$$= \frac{1{\cdot}01 - 0{\cdot}7}{1{\cdot}01 - 0{\cdot}21} = \frac{0{\cdot}31}{0{\cdot}8} = 0{\cdot}387$$

Problem 14. A sample of sand is taken from a natural deposit by means of a sampling cylinder and the following data recorded :—

Volume of cylinder, 382 c.c.; weight of sample in natural state, 707 grs.; weight of sample when dried, 664 grs.; volume of sample when dried and rammed tightly into tube, 334 c.c.; volume of sample when dried and placed loosely into tube, 493 c.c.; and specific gravity of solids, 2·62.

What is (*a*) the relative humidity and (*b*) the degree of density of the deposit ?

(*a*) Voids ratio, $e = \frac{\text{Volume of voids}}{\text{Volume of solids}}$

$$= \frac{\text{Vol. of air} + \text{Vol. of water} + \text{Vol. of solids}}{\text{Vol. of solids}} - 1$$

$$= \frac{382 \times 2{\cdot}62}{664} - 1 = 0{\cdot}51$$

$$\text{Volume of solids} = \frac{\text{Wgt. of solids}}{\text{Specific gravity}} = \frac{664}{2{\cdot}62}$$

Relative humidity

$$= \frac{\text{Wgt. of water per unit volume of solid material}}{\text{Voids ratio}}$$

$$= \frac{\frac{(707 - 664) \times 2{\cdot}62}{664}}{0{\cdot}51} = \frac{43 \times 2{\cdot}64}{664 \times 0{\cdot}51} = \frac{112{\cdot}66}{338{\cdot}64} = 0{\cdot}332$$

(*b*) Relative density

$$= \frac{\text{Max. voids ratio} - \text{Natural voids ratio}}{\text{Max. voids ratio} - \text{Minimum voids ratio}}$$

Maximum—

$$e = \frac{493 \times 2{\cdot}62}{664} - 1 = \frac{1291{\cdot}66}{664} - 1 = 0{\cdot}95$$

Minimum—

$$e = \frac{334 \times 2{\cdot}62}{664} - 1 = \frac{875{\cdot}08}{664} - 1 = 0{\cdot}32$$

Relative density—

$$= \frac{0{\cdot}95 - 0{\cdot}51}{0{\cdot}95 - 0{\cdot}32} = \frac{0{\cdot}44}{0{\cdot}63} = 0{\cdot}698$$

Problem 15. A fully saturated clay sample has a volume of 185 c.c. and weighs 331 grs. If the absolute density is 2·67, find the voids ratio, porosity, moisture content and unit weight in lb. per cu. ft.

(*a*) Voids ratio.

Weight of water + weight of soil = 331 grs.
Volume of water + volume of soil = 185 c.c.
In metric units, weight of water = volume of water,
∴ 2·67 × vol. of soil − vol. of soil = 331 − 185 = 146.

$$\therefore \text{ Volume of soil} = \frac{146}{2{\cdot}67 - 1} = 87{\cdot}4 \text{ c.c.}$$

$$\therefore \text{ Volume of water} = 185 - 87{\cdot}4 = 97{\cdot}6 \text{ c.c.}$$

$$\therefore \text{ Voids ratio,} \quad e = \frac{97{\cdot}6}{87{\cdot}4} = 1{\cdot}116 = 111{\cdot}6\%$$

(*b*) Porosity, $n = \frac{e}{1 + e} = \frac{1{\cdot}116}{1 + 1{\cdot}116} = 0{\cdot}53 = 53\%$

(*c*) Moisture content, $W = \frac{e}{\rho}$ from equation (4)

$$= \frac{1{\cdot}116}{2{\cdot}67} = 0{\cdot}418 = 41{\cdot}8\%$$

(*d*) Unit weight—

$$= \rho \frac{1 + W}{1 + e} = 2{\cdot}67 \frac{1 + 0{\cdot}418}{1 + 1{\cdot}116} = 1{\cdot}79 \text{ grs. per c.c.}$$

$$= 1{\cdot}79 \times 62{\cdot}4 \qquad = 111{\cdot}7 \text{ lb. per cu. ft.}$$

The determination of soil moisture and density by nuclear radiation

As a result of research grants made by the Saskatchewan Research Council and the Prairie Roadbuilders Section of the Canadian Construction Association, investigations have been made on the determination of soil moisture and density by nuclear radiations, and in 1952 a Paper was presented to the American Society of Testing Materials in Cleveland, Ohio, by Messrs. D. A. Lane, B. B. Torchinsky and J. W. T. Spinks on the development of a meter which can be used for this purpose. As far as can be ascertained, no research of a similar nature has been undertaken in the British Isles up to the present.

(*a*) *The determination of soil moisture.*

The moisture meter is based on the fast neutron bombardment of the soil, which results in the reflection of slow neutrons and their action on a detector of indium foil. The apparatus consists of a cone-shaped head which is lowered into a 2-in. diameter vertical tube sunk into the soil. Attached to this head is a capped aluminium cylinder about 11 ins. in length. A neutron source is placed in position by means of electro-magnets within the cylinder, and a holder containing indium foil is lowered into its seating by means of a cord, thus positioning the indium foil around the neutron source. (Fig. 2*a*.)

The fast neutrons emitted by the neutron source lose their energy by collision with the hydrogen atoms in the water molecules in the soil and, therefore, become slow neutrons which are absorbed by the stable indium foil causing a change to radioactive indium. The slow neutrons are more readily absorbed than the fast neutrons and the activity induced in the foil during a given period depends upon the amount of moisture in the soil. An equation has been evolved which relates the amount of activity of the foil to the soil moisture content. The foil is exposed for a definite period, withdrawn in its holder and placed round the Geiger tube of a portable meter, and a reading is taken at the termination of a further definite period of time. In practice, a specific procedure of a ten-minute exposure with a further one-minute delay before measurement, has been chosen as a standard routine. An initial reading of the foil should be taken before it is used and an additional five or six identical foils are required for use, so that a reasonable period of time can elapse before their re-use. Generally, for practical purposes, the zone of influence can be considered to be a sphere of six-inch radius.

(*b*) *The determination of soil density.*

The density meter is based on the amount of gamma-ray absorption of the soil which is proportional to its density. The same neutron source is utilised as before in conjunction with the Geiger tube of the portable meter, but the cylinder is larger, being about 20 ins. long with a lead shield placed between it and the

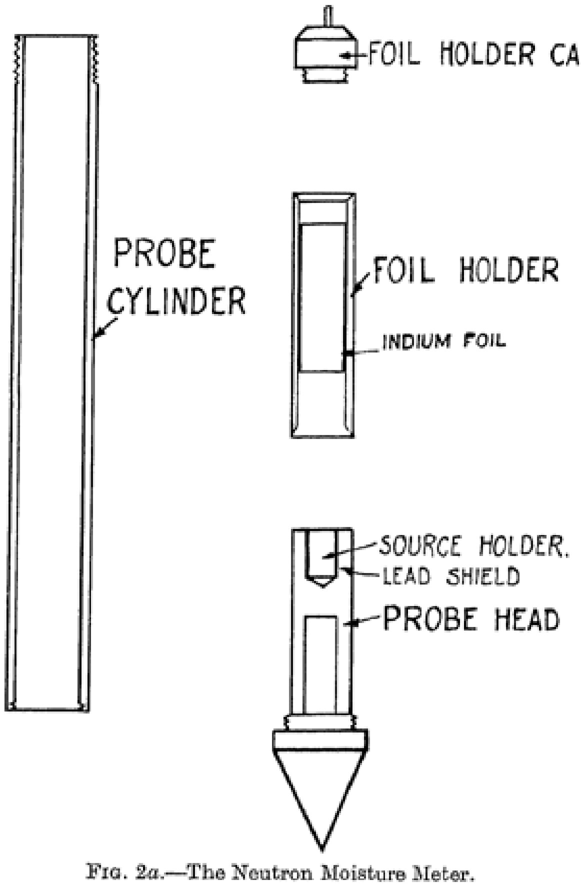


FIG. 2*a*.—The Neutron Moisture Meter.

neutron source as a protection to direct gamma radiation. Thus the only rays which can strike the tube are those through the soil. The tube is connected to the portable rate meter which indicates the average rate at which gamma rays reach the tube, and this value can be calibrated against wet density. It is essential, however, that the relative positions of the source and the Geiger

tube are constant throughout the tests in order to produce accurate and comparable calibration curves.

A number of field experiments have been carried out using aluminium tubes and steel pipes, and it is only necessary to adjust the calibration curves according to the material used. These results were checked with oven-drying methods and all determinations for moisture content agreed within at least 3 per cent., although most of the results were within 2 per cent.

Tests in ground which was flooded gave similarly effective results, but where soils have an organic composition some variation does occur. The presence of organic material affects the meter in the same way as if water were present and a correction factor must be introduced dependent upon the soil type, and further experimental work is taking place in this field.

Another possible use for the neutron meter is to determine the asphaltic content of a bituminous pavement composition. The activation of the meter is similar to that for determining the water content and a compensating calibration curve can be developed to indicate the asphaltic percentage present.

The use of the neutron and density meters necessitates certain precautions being taken by the operator. An electro-magnet mounted on a 3-ft. length of $\frac{5}{16}$-inch diameter rod projecting from an ordinary flashlight case facilitates the safe handling of the neutron source. This source is stored in a lead sheath weighing about 75 lb. When the neutron source is placed in position the operator must not stand directly over it and he must carry a film monitor indicating the exposure to gamma rays, so that he may keep within the allowable margin.

CHAPTER II

SOIL PROPERTIES—LIMITS OF CONSISTENCY, PERMEABILITY AND SHEAR STRENGTH

Limits of Consistency

The limits of consistency of a soil mass, especially clays, are divided for convenience into stages. Assuming the mass is in a liquid state, on drying out it becomes stiffer and can be moulded with the fingers. The soil is now in a plastic state, and the moisture content when this change occurs is known as the liquid limit.

On further loss of moisture the soil becomes semi-solid and crumbles when worked with the fingers, and this lower moisture content is termed the plastic limit. Further loss of moisture causes the soil to become solid, and this limit is called the shrinkage limit.

Experiments for determining the limits of consistency are carried out only on particles passing a B.S. No. 36 sieve, and when reporting on a sample the percentage of these particles of the whole sample must be stated.

Liquid Limit.

The liquid limit is the moisture content at which the soil is sufficiently fluid to flow a specified amount when lightly jarred twenty-five times. The apparatus in which the soil is made to flow is a standard Casagrande Liquid Limit apparatus, which consists of a brass dish and cam mounted on a hard rubber base, as shown in Fig. 3. As the small handle rotating the cam is turned at a rate of two rotations per second, the dish falls through a distance of 1 cm. per rotation.

A sample of soil is placed in the dish and levelled off so that it is 1 cm. thick in the centre of the bowl. A groove 11 mm. wide at the surface and 2 mm. at the bottom of the sample is made with a scraper, and the number of blows required to cause the 2 mm. gap to close along $\frac{1}{2}$ inch is recorded.

When performing the experiment it is usual to take two series of readings for the moisture content necessary to close the standard-size groove in the soil sample. One series should be taken with 5 to 15 blows, and the other series with 20 to 30 blows. The moisture content is plotted arithmetically against the number of blows required to close the groove plotted logarithmically, and the moisture content corresponding to 25 blows is the liquid limit. (Problem 19 will illustrate this experiment.)

It will be apparent that the nearer the natural moisture content is to the liquid limit the softer is the soil. Silty clays have a low liquid limit, whilst colloidal clays have a high liquid limit, possibly exceeding 100.

Plastic Limit.

The plastic limit is the lower limit of plasticity, and is the lower moisture content at which a thread of soil can be rolled down without breaking until it is $\frac{1}{8}$ in. in diameter.

The plastic limit tends to be increased, sometimes to a considerable extent, by a small percentage of organic matter in the soil.

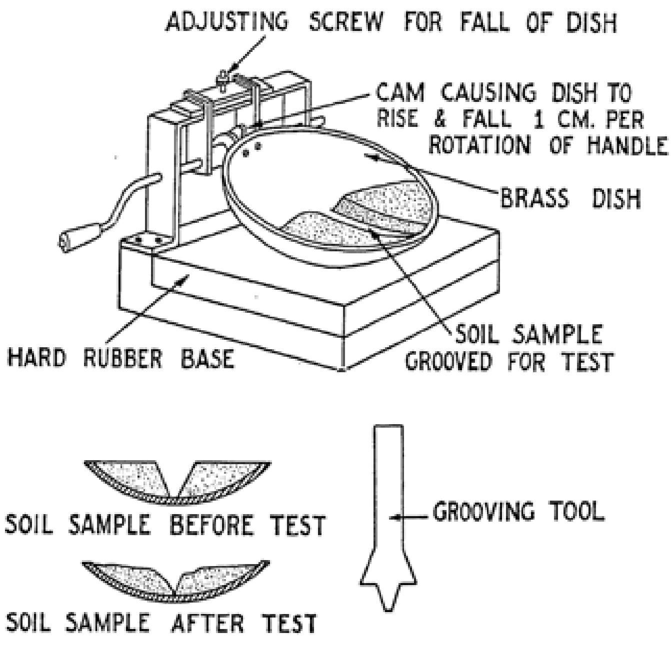


FIG. 3.—Casagrande Liquid Limit Apparatus.

Shrinkage Limit.

The shrinkage limit is defined as the moisture content at which a soil sample ceases to lose volume but continues to lose weight.

Shrinkage limit

$$=\frac{\text{Voids ratio}}{\text{Specific gravity of particles}}=\frac{1}{s_m}-\frac{1}{\rho} \quad . \quad . \quad (12)$$

where s_m is the specific mass gravity or apparent specific gravity, and ρ is the specific gravity of the particles. This formula is derived from the fact that in the dry state the moisture content is zero and the volume is constant.

Shrinkage effects are greatest in clays with a high liquid limit—that is, clays with a high proportion of colloidal particles, which may be recognised in winter by their gluey and sticky nature, and in summer by the appearance of shrinkage cracks.

Plasticity Index.

The plasticity index of a soil = Liquid limit − Plastic limit.

The more plastic the type of soil, the greater is the plasticity index, thus the plasticity index of a coarse sand is close to zero.

Typical values for the liquid limit, plastic limit and plasticity index are as follows :—

London clay.	Liquid limit 75.	Plastic limit 25.	Plasticity index 50.
Silty clay.	,, ,, 48.	,, ,, 24.	,, ,, 24.
Silt.	,, ,, 36.	,, ,, 20.	,, ,, 16.

Liquidity Index.

The consistency of a soil at its natural moisture content can be conveniently expressed by its liquidity index defined thus :—

Liquidity index

$$= 100 \times \frac{(\text{Natural moisture content} - \text{Plastic limit})}{\text{Liquid limit} - \text{Plastic limit}} \quad . \quad (13)$$

Problem 16. A sample of clay has a liquid limit of 50 per cent. and a plastic limit of 25 per cent. What type of clay would this information indicate ? How would it differ from another sample of clay with a liquid limit of 52 per cent. and a plastic limit of 40 per cent. ?

The first sample has a plasticity index =

$$\text{Liquid limit} - \text{Plastic limit} = 50 - 25 = 25,$$

and the second sample has a plasticity index = $52 - 40 = 12$.

The first sample has a liquid limit of 50 per cent., which is not high, and indicates a considerable proportion of silt and sand. The plasticity index of 25 confirms this, and in all probability the soil would be a silty clay.

The plastic limit of 25 per cent. in the first sample is low, thus indicating the presence of a high proportion of flaky or laminar mineral particles with very little, if any, organic matter. The second sample has a high plastic limit of 40 per cent., which suggests the absence of laminar crystals and the presence of organic matter to a considerable degree.

Problem 17. A sample of clay has an absolute density of 2·79. In an undisturbed state the voids ratio is 75 per cent., but a remoulded sample has a lower value of 56 per cent. Find the shrinkage limits in each case and state the conclusions which may be drawn from this data.

From equation (12) the shrinkage limit of the undisturbed sample

$$= \frac{0{\cdot}75}{2{\cdot}79} = 0{\cdot}27 = 27\%$$

The shrinkage limit of the remoulded sample

$$= \frac{0{\cdot}56}{2{\cdot}79} = 0{\cdot}20 = 20\%$$

It is apparent that in the remoulded sample the voids are less than in the natural undisturbed state of the clay and less moisture is required to completely fill the pores. It can be deduced that the compressibility of the clay in its natural state is high. This soil as excavated would be unsuitable for the construction of an embankment, but would be satisfactory if used in a remoulded condition, which could be effected by tipping in shallow layers and consolidating with rollers.

Problem 18. A fully saturated clay has a moisture content of 45 per cent. and an apparent specific gravity of 1·95, but after drying out the specific gravity is 1·79. Find the absolute density and the shrinkage limit for this clay.

From equation (4)

Voids ratio, $e = \rho W = 0{\cdot}45\,\rho$

From equation (7)

Saturated density $= 1{\cdot}95 = \dfrac{\rho + e}{1 + e}$

Absolute density, $\rho = 3{\cdot}22$

and $e = 1{\cdot}449 = 144{\cdot}9\%$

From equation (12)

$$\text{Shrinkage limit} = \frac{1}{s_m} - \frac{1}{\rho}$$

$$= \frac{1}{1{\cdot}79} - \frac{1}{3{\cdot}22}$$

$$= 0{\cdot}558 - 0{\cdot}3105 = 0{\cdot}2475 = 24{\cdot}75\%$$

(*Note.*—S_m is the specific gravity of the clay in the completely dry state.)

Problem 19. A test is carried out on a clay sample with a Casagrande liquid limit apparatus and ranges of three readings for two groups are tabulated below. The first group consists of readings chosen between 5 and 15 blows, and the second group between 20 and 35 blows. From the straight line joining these results on a graph determine the liquid limit.

Reading No.	Number of blows.	Moisture content.	Reading No.	Number of blows.	Moisture content.
Group 1	6·5 9 13	0·53 0·502 0·463	Group 2	27 29 33	0·355 0·342 0·325

(*Note.*—The first reading of 6·5 blows indicates that the sixth blow closed the groove in the soil sample along $\frac{1}{4}$ in., whilst the seventh blow brought the two sides together along $\frac{3}{4}$ in.)

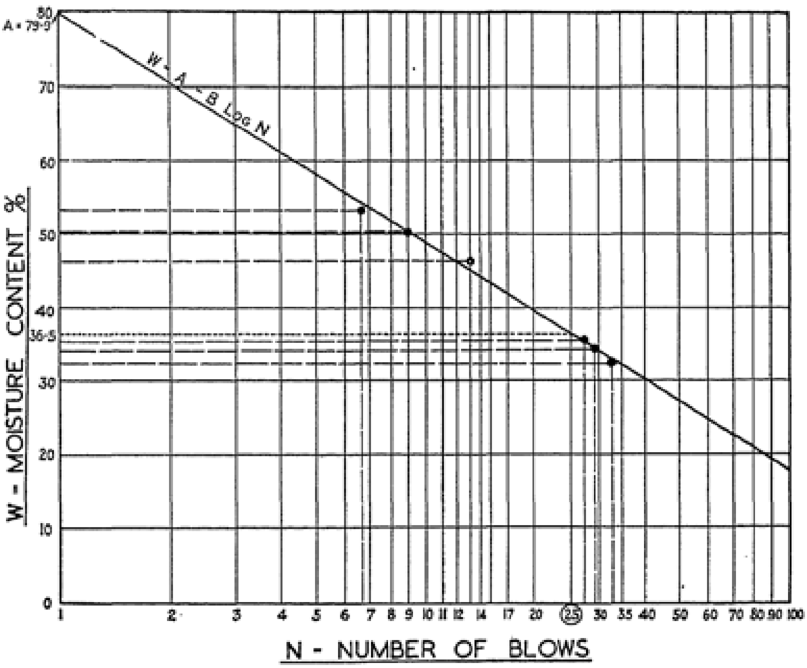


FIG. 4.—Liquid Limit Graph.

C

In Fig. 4 the moisture content, W, is plotted vertically to an arithmetic scale, and the number of blows, N, are plotted horizontally to a logarithmic scale. The various points obtained are joined by a straight line, and the moisture content corresponding to 25 blows is the liquid limit. In the soil sample tested the liquid limit is 0·365, or 36·5 per cent.

(*Note.*—The equation for this straight line is of the form $W = A - B \log N$, where A and B are constants. From inspection of Fig. 4, A is the intercept on the vertical axis where $W = 0{\cdot}799$. Thus A is the moisture content at which the groove in the soil sample will close along $\frac{1}{2}$ in. with one blow.)

From simultaneous equations for the straight-line graph :—

$$\left.\begin{aligned} W_1 &= A - B \log N_1 \\ W_2 &= A - B \log N_2 \end{aligned}\right\}$$

$$B = \frac{W_2 - W_1}{\log N_1 - \log N_2} = \tan \phi$$

where ϕ is the angle between the straight-line graph and the horizontal axis.

In the case under consideration $B = 0{\cdot}3073$ and $\phi = 17° 5'$.

(The constant B is known as the " Flow index " of the soil.)

Permeability

The passage of water through soils is one of the most important properties, and is known as permeability. The degree of permeability of a soil is expressed as the coefficient of permeability, k.

Impervious soils are desirable in the construction of earth dams for reservoirs and canals, whilst permeable soils are important in the construction of road and railway embankments, and road and railway formations. In foundations good permeability is an asset, since the settlement of a structure depends on the rate of squeezing out of moisture.

According to Darcy, when the hydraulic gradient is equal to or less than unity, the discharge velocity, v, under the action of an hydraulic gradient, i, is given by the equation :

$$v = ki \qquad \text{. . . . . . . (14)}$$

The total discharge, q, through an area, A, during an interval of time, t, will be given by :

$$q = Akit \qquad \text{. . . . . . . (15)}$$

Test results obtained by Tschebotareff are shown in Table 3 :—

TABLE 3.

Permeability of Soils.

Type of soil.	k in feet per day.	k in cms. per minute.
Gravel	23,600 to 2,360	5,000 to 50
Sand	2,360 to 2·4	50 to 0·05
Fine sand and silt	2·4 to 0·00024	0·05 to 0·000005
Clay.	0·00024 to nil	0·000005 to nil

The condition of a quicksand exists when the upward flow through the soil is sufficient to float the soil grains, and the critical hydraulic gradient for this condition is given by the equation :

$$i = \frac{\rho - 1}{1 + e} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

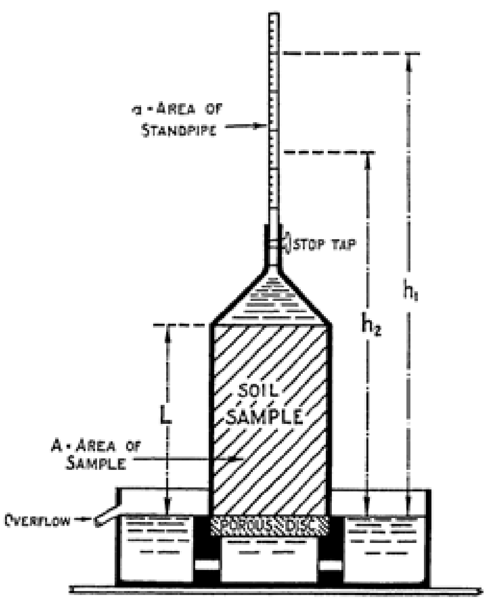


FIG. 5.—Variable-Head Permeameter.

Failure in earth dams and embankments known as "piping" is due to this occurrence.

The coefficient of permeability is obtained in the laboratory by means of a constant-head permeameter or falling-head permeameter.

A falling-head permeameter is illustrated by a diagram in Fig. 5. In this apparatus a sample of thickness, L, is contained in a vessel

which is placed on a porous disc and rests in a tank of water, so that the water level cannot rise above the lower surface of the soil sample. The hollow screw top of the vessel is provided with a standpipe and stopcock. The top is screwed in position, and water is introduced through the standpipe. The measurement of the percolation of water through the sample enables the coefficient of permeability to be obtained. The passage of water between the sample and the sides of the container will influence the results, and a paraffin or bentonite layer inside the vessel is necessary.

For a constant-head permeameter, Darcy's equation (15) is applicable and

$$k = \frac{q}{Ait}$$

For a falling-head permeameter, Darcy's equation becomes

$$k = \frac{2{\cdot}303aL}{A(t_2 - t_1)} \log \frac{h_1}{h_2}$$

where a = area of standpipe,
L = thickness of sample,
h_1 = head of water at t_1,
h_2 = head of water at t_2.
A = area of sample,
$t_2 - t_1$ = time interval for experiment.

Problem 20. What is the critical hydraulic gradient which would cause a quicksand condition to exist in a soil consisting of sand grains having a specific gravity of 2·75 and a voids ratio of 0·95?

From equation (16), the critical hydraulic gradient,

$$i = \frac{\rho - 1}{1 + e} = \frac{2{\cdot}75 - 1}{1 + 0{\cdot}95} = 0{\cdot}9$$

Problem 21. Water is percolating through the earth fill of a coffer-dam 50 ft. long and 25 ft. thick, the depth of water being 16 ft. over an impervious layer of clay. Assuming the sheet piling is not sufficiently tight to retard the flow of water, calculate the total amount of water that will flow through the entire coffer-dam per day if (i) the earth fill is a silty clay with a coefficient of permeability of 0·00022 ft. per day, and (ii) the earth fill is a silty sand with $k = 16{\cdot}5$ ft. per day.

From equation (15), total discharge, q = Akit

$$\left.\begin{aligned}\text{(i) where } A &= 50' \times 16' = 800 \text{ sq. ft.}\\ k &= 0{\cdot}00022 \text{ ft. per day.}\\ i &= \frac{h}{L} = \frac{16}{25} = 0{\cdot}64.\\ t &= 1 \text{ day.}\end{aligned}\right\}$$

$$\begin{aligned}q &= 800 \times 0{\cdot}00022 \times 0{\cdot}64 \times 1\\ &= 0{\cdot}113 \text{ cu. ft. per day where the earth fill is a silty clay.}\end{aligned}$$

$$\left.\begin{aligned}&\text{(ii) where } k = 16{\cdot}5 \text{ ft. per day and}\\ &\qquad A, i \text{ and } t \text{ are as before.}\end{aligned}\right\}$$

$$\begin{aligned}q &= 800 \times 16{\cdot}5 \times 0{\cdot}64 \times 1\\ &= 9{,}438 \text{ cu. ft. per day where the earth fill is a silty sand.}\end{aligned}$$

Problem 22. In a constant-head permeameter 3·42 cu. ins. of water passed through a cylindrical soil sample 5 ins. thick and 3 ins. in diameter during a period of 1·5 minutes with an effective head of 1 ft. What is the coefficient of permeability in feet per day?

From Darcy's equation (15), $q = Akit$ or $k = \dfrac{q}{Ait}$

$$\left.\begin{aligned}q &= 3{\cdot}42 \text{ cu. ins.} = \frac{3{\cdot}42}{12^3} \text{ cu. ft.}\\ t &= 1{\cdot}5 \text{ mins.} \quad = \frac{24 \times 60^2}{90} \text{ day.}\\ i &= \frac{h}{L} = \frac{12}{5} \quad = 2{\cdot}4.\\ A &= 3{\cdot}142 \times \left(\frac{3}{2}\right)^2 \text{ sq. in.} = \frac{7{\cdot}07}{12^2} \text{ sq. ft.}\end{aligned}\right\}$$

$$\begin{aligned}k &= \frac{3{\cdot}42 \times 12^2 \times 60^2 \times 24}{12^3 \times 7{\cdot}07 \times 90 \times 2{\cdot}4}\\ &= 16{\cdot}13 \text{ ft. per day.}\end{aligned}$$

Problem 23. In a variable-head permeameter the initial head of 15 ins. drops to 7·36 ins. in 1·5 minutes, the diameter of the standpipe being 1 in. The sample is 6 ins. thick and 3·5 ins. in diameter. Determine the coefficient of permeability in feet per day.

A diagram of the apparatus is given in Fig. 5.

For the variable-head permeameter, Darcy's law becomes

$$k = \frac{2{\cdot}303aL}{A(t_2 - t_1)} \log \frac{h_1}{h_2}$$

where a $=$ Area of standpipe $= \dfrac{3{\cdot}142 \times 1}{12^2 \times 4}$ sq. ft.

L $=$ Thickness of sample $= 6'' = 0{\cdot}5'$

A $=$ Area of sample $= \dfrac{3{\cdot}142 \times 3{\cdot}5^2}{4 \times 12^2}$ sq. ft.

$t_2 - t_1 =$ Time interval during experiment

$$= 90 \text{ secs.} = \frac{90}{60^2 \times 24} \text{ days}$$

$h_1 =$ Head of water at $t_1 = \dfrac{15}{12}$ ft.

$h_2 =$ Head of water at $t_2 = \dfrac{7{\cdot}36}{12}$ ft.

$$k = \frac{2{\cdot}303 \times 3{\cdot}142 \times 1 \times 4 \times 12^2 \times 60^2 \times 24}{4 \times 12^2 \times 2 \times 3{\cdot}142 \times 3{\cdot}5^2 \times 90} \times \log \frac{15}{7{\cdot}36}$$
$$= 27{\cdot}94 \text{ ft. per day.}$$

Problem 24. Excavation is being carried out in a sandy soil that has a porosity of 45 per cent. and an absolute density of 2·7. What is the critical gradient which would cause a "running-sand" condition?

From equation (3)

$$\text{Voids ratio, } e = \frac{n}{1-n} = \frac{0{\cdot}45}{1 - 0{\cdot}4} = \frac{0{\cdot}45}{0{\cdot}55}$$

From equation (16)

$$\text{Critical gradient, } i = \frac{\rho - 1}{1 + e}$$
$$= \frac{2{\cdot}7 - 1}{1 + \dfrac{0{\cdot}45}{0{\cdot}55}} = \frac{1{\cdot}7}{1{\cdot}82}$$
$$= 0{\cdot}93$$

Shear Strength

In order to compute the resistance of a soil against failure, it is necessary to carry out experiments on a sample to ascertain the shear strength. The apparatus used is known as a shear-box, and measurement is made of the force required to shear the sample in half horizontally. With this apparatus it is possible to execute immediate shear tests as well as tests with vertical loading which allow the samples to be fully consolidated. The introduction of

porous stones in the shear-box enables experiments to be made with completely saturated samples.

Shear-box tests are performed in the laboratory, but a useful type of portable apparatus which may be utilised for carrying out tests at the site is the Building Research Station unconfined compression apparatus as shown in Fig. 6. This apparatus consists of two metal platforms, one being fixed, whilst the other is capable of being moved vertically along guides and is supported

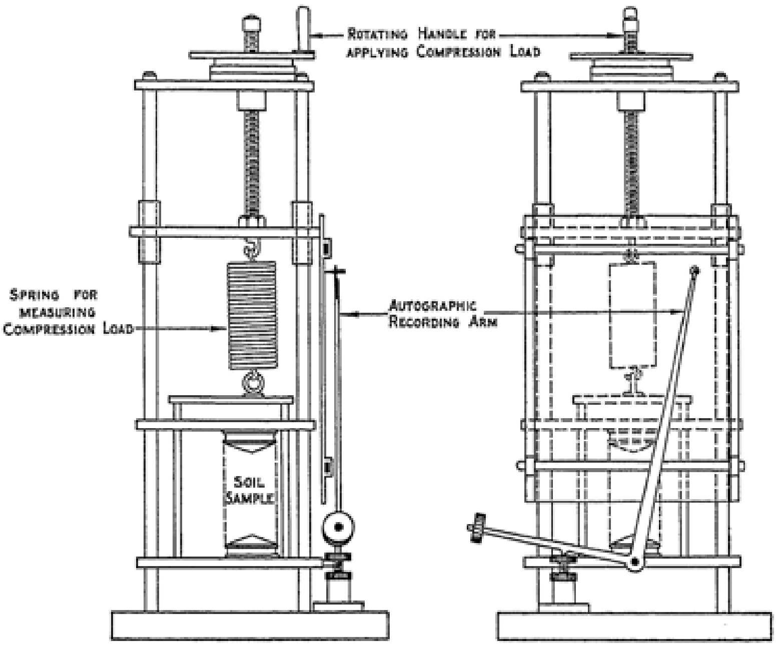


FIG. 6.—Unconfined Compression Apparatus.

in position by a coil spring. By rotating the handle at the top of the apparatus, the screw thread causes the movable platform to be raised until a standard-size soil sample is held between the two coned end-pieces attached to the platforms. An undisturbed cylindrical soil sample is placed in position so that there is no tension in the coil spring, and the autographic recording arm is adjusted. For the test the handle is rotated at an even rate until the soil specimen fails either by bulging and collapsing, or by shearing at an angle to the vertical axis. The compressive stress required to cause failure in a clay sample is at least twice the

value of the shear stress. The apparatus cannot be used for sands or wet clays when the samples are not sufficiently firm to be handled and placed in the machine.

According to Coulomb's formula, the shearing resistance per unit area,

$$s = c + \sigma \tan \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where c = apparent cohesion,
σ = normal pressure, and
ϕ = angle of internal friction.

In an immediate shear test with no consolidation of the sample (that is provided no water can escape from the sample) the angle

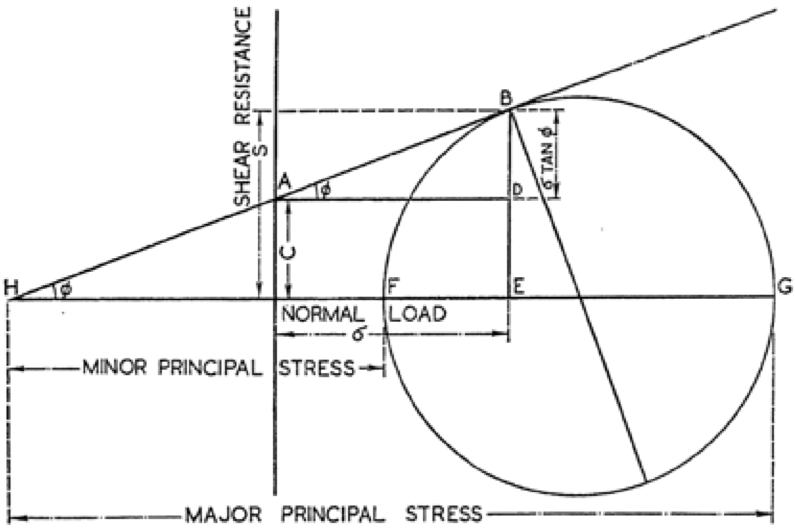


Fig. 7.—Mohr's Circle.

of internal friction is zero and the shear resistance is equal to the apparent cohesion per unit area. It may be noted that soft and silty clays act in this manner as purely cohesive or frictionless soils.

Shear tests on clay samples with full consolidation allowed to take place are carried out with a series of three or more results, and by plotting a graph of normal loading to shear resistance enables c and $\sigma \tan \phi$ to be determined. (See Problem 26.)

Full consolidation of sands results in most cases to

$$s = \sigma \tan \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The Coulomb formula may be graphically expressed in diagram form as in Fig. 7. The shear stress, s, is plotted vertically, and

the normal loading, σ, plotted horizontally. Assuming the line A to B joins the points obtained from a series of tests on a soil sample, it will be evident that the shear stress, s, is composed of two parts, BD and DE. DE is the intercept of BA on the vertical axis and represents the apparent cohesion, c, whilst BD equals $\sigma \tan \phi$, where ϕ is the angle AB makes with the horizontal axis. Thus, s is equal to $c + \sigma \tan \phi$.

With a centre on the horizontal axis draw a circle with tangent AB and tangential at B. This circle is known as Mohr's circle, and enables the principal stresses in the sample to be obtained. If the circle cuts the horizontal axis at F and at G, and BA produced cuts the same axis at H, then the major principal stress is HG and the minor principal stress is HF, measured from the y axis.

Problem 25. In a B.R.S. type unconfined compression apparatus a cylindrical sample of sandy clay, $3\frac{1}{2}$ ins. long and $1\frac{1}{2}$ ins. in diameter, fails under a load of 16·5 lb. Estimate the shearing resistance of this soil. What would be the shearing resistance if a shortening of the sample by $\frac{3}{8}$ in. is taken into account?

Assuming the compressive stress is double the shear stress, then

$$s = \frac{16{\cdot}5 \times 144}{2 \times 3{\cdot}142 \times 0{\cdot}75^2} = 655 \text{ lb. per sq. ft.}$$

Taking into account the shortening of the sample, then

$$\text{compressive stress} = 2s = \frac{P}{A}(1 - d)$$

where P is the load, A is the original cross section and

$$d = \frac{h - h_1}{h}$$

Therefore, shear strength,

$$s = \frac{16{\cdot}5 \times 144}{2 \times 3{\cdot}142 \times 0{\cdot}75 \times 0{\cdot}75} \times \left(1 - \frac{\frac{3}{8}}{3{\cdot}5}\right)$$
$$= 600 \text{ lb. per sq. ft.}$$

Problem 26. The following series of readings were taken during a shear-box test on a sandy clay :—

Horizontal shear force,			18 lb.	Vertical loading,	10 lb.
,,	,,	,,	22 lb.	,, ,,	20 lb.
,,	,,	,,	27 lb.	,, ,,	30 lb.
,,	,,	,,	33 lb.	,, ,,	40 lb.

The shear box is 2 ins. square and 3 ins. deep. Plot these results in the form of a graph and determine the apparent cohesion and angle of internal friction for this soil.

The graph is plotted as in Fig. 8, and from inspection the cohesive force, c, is 13 lb., or 468 lb. per sq. ft., and the angle of internal friction, ϕ is 26°.

(Check.) From simultaneous equations of the first and fourth readings :

$$\left.\begin{aligned} 18 &= c + 10 \tan \phi \\ 33 &= c + 40 \tan \phi \end{aligned}\right\} \begin{aligned} &15 = 30 \tan \phi \\ &\therefore \tan \phi = 0{\cdot}5 \text{ or } \phi = 26\tfrac{1}{2}° \end{aligned}$$

$$\begin{aligned} c &= 18 - (10 \times 0{\cdot}5) \\ &= 13 \text{ lb. or } 468 \text{ lb. per sq. ft.} \end{aligned}$$

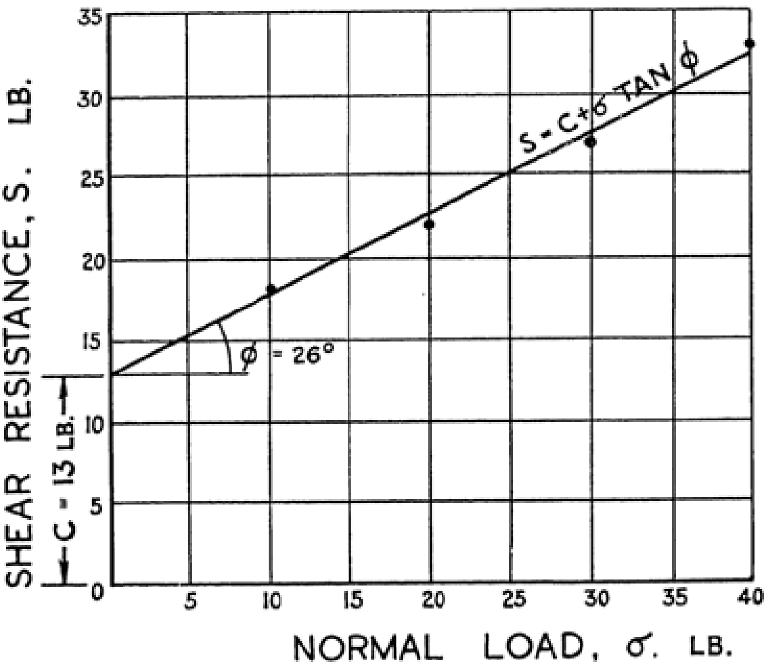


FIG. 8.—Shear-box Test.

Problem 27. During a fully consolidated shear-box test on a sample of sand, a shearing stress of 800 lb. per sq. ft. produces failure with a vertical load of 1,750 lb. per sq. ft. Determine by means of Mohr's circle the maximum and minimum principal stresses.

The diagram with Mohr's circle is indicated in Fig. 9.

From the figure the maximum principal stress amounts to 2,980 lb. per sq. ft. and the minimum principal stress is 1,240 lb.

per sq. ft. As the soil is a sand and full consolidation is allowed, equation (18) is applicable, therefore,

$$s = \sigma \tan \phi$$
$$800 = 1750 \tan \phi$$
$$\therefore \tan \phi = 0{\cdot}4572 \text{ or } \phi = 24\tfrac{1}{2}^\circ$$

(*Note.*—If OC is the bisector of angle AOB ($= 90^\circ - \phi$), then OC is the direction of the major principal stress and the angle COB is $\frac{90^\circ - \phi}{2}$. Similarly, if OE is the bisector of angle AOD ($= 90^\circ + \phi$), then OE is the direction of the minor principal stress and the angle DOE with the horizontal axis is $\frac{90^\circ + \phi}{2}$.)

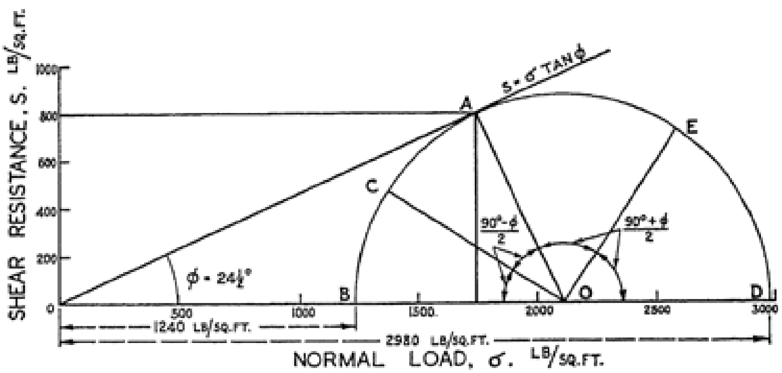


FIG. 9.—Shear-box Tests.

Problem 28. A series of shear-box tests are carried out on samples of three different types of soil, and the results are tabulated below :—

Designation of test.	Horizontal shearing force.	Vertical loading.
Soil A.—Test 1 . . .	18·1 lb.	6 lb.
Test 2 . . .	19·0 ,,	9 ,,
Test 3 . . .	20·3 ,,	15 ,,
Soil B.—Test 1 . . .	12·5 lb.	20 lb.
Test 2 . . .	18·7 ,,	30 ,,
Test 3 . . .	28·1 ,,	45 ,,
Soil C.—Test 1 . . .	11·5 lb.	25 lb.
Test 2 . . .	11·8 ,,	40 ,,
Test 3 . . .	12·0 ,,	50 ,,

The area of the sample sheared is 4 sq. ins. Plot the above results, and from the straight-line graphs obtain the apparent cohesion and angle of internal friction. What is the probable type of soil in each sample ?

Soil A :—The results are plotted as indicated in Fig. 10. From inspection of the graph, the apparent cohesion is 16·5 lb., or 594 lb. per sq. ft., and the angle of internal friction is 14°.

$$\text{(Check: } s = 18{\cdot}1 = c + 6\tan\phi \atop 20{\cdot}3 = c + 15\tan\phi \left\{ \begin{array}{l} \operatorname{Tan}\phi = 0{\cdot}244 \text{ or } \phi = 13^\circ\, 43'. \\ c = 18{\cdot}1 - (6 \times 0{\cdot}244) = 16{\cdot}636 \text{ lb.} \\ \text{or } c = 599 \text{ lb. per sq. ft.)} \end{array}\right.$$

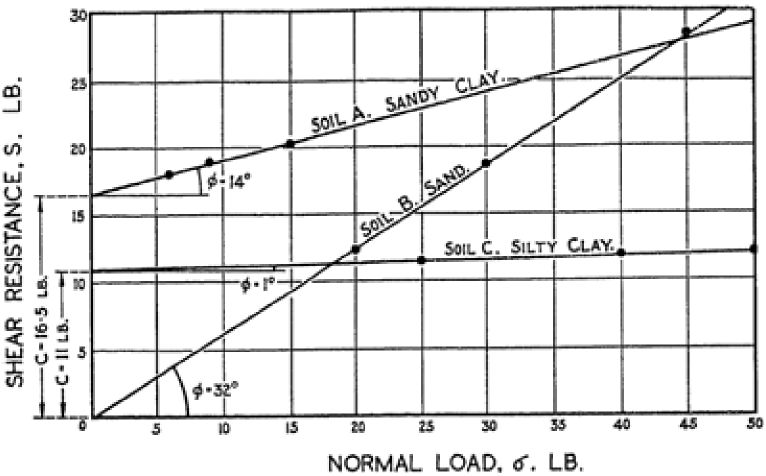


FIG. 10.—Shear-box Tests.

Soil sample A has a considerable angle of internal friction in addition to cohesion, and is probably a sandy clay.

Soil B. From inspection of Fig. 10, it is apparent that there is no cohesive force and the angle of internal friction is 32°.

$$\text{(Check: } s = 12{\cdot}5 = c + 20\tan\phi \atop 28{\cdot}1 = c + 45\tan\phi \left\{ \begin{array}{l} \operatorname{Tan}\phi = 0{\cdot}624 \text{ or } \phi = 32^\circ. \\ c = 12{\cdot}5 - (20 \times 0{\cdot}624) = 0 \\ \text{or } c \text{ is equal to zero.)} \end{array}\right.$$

Soil sample B has a high angle of internal friction but no cohesion, and is probably a sand.

Soil C. From Fig. 10 it will be seen that the apparent cohesion

is 11 lb., or 369 lb. per sq. ft. The angle of internal friction is very small, probably about 1°.

$$\text{(Check : } s = 11{\cdot}5 = c + 25 \tan\phi \qquad 12{\cdot}0 = c + 50 \tan\phi \left\{ \begin{array}{l} \text{Tan}\,\phi = 0{\cdot}02 \text{ or } \phi = 1^\circ\,10'. \\ c = 12 - (50 \times 0{\cdot}02) = 11 \text{ lb.} \\ \qquad \text{or } c = 396 \text{ lb. per sq. ft.)} \end{array} \right.$$

Soil sample C has a small angle of internal friction with an appreciable cohesive force, and the soil is probably a silty clay.

Note on the Permeability of Fine-grained Soils. (See page 21)

In the case of fine-grained soils it is sometimes found that the use of ordinary water produces erratic results when determining k. In such cases it may be necessary to use distilled de-aerated water; when this is done the air in the water is removed before it can reach the sample and is thus prevented from clinging to the fine particles and thereby restricting the flow through the sample. The de-aeration is carried out by applying a high vacuum to the still and condenser during distillation.

Note on the Trixial Compression Test

A more useful apparatus for the determination of shear strength than the shear box is the triaxial compression machine, in which a cylindrical soil sample is subjected to an axial load. The rate of application of the load is variable between wide limits, whilst at the same time the sample is laterally constrained by a liquid pressure which can be adjusted so that it will represent the restraining action of the surrounding soil to the sample before it was taken. It is essential that the ingress or egress of water from the sample be strictly controlled, and it is for this reason that the sample is inserted in a fine rubber sheath. By this means the liquid providing the lateral stress is isolated from the soil sample, whilst the moisture in the latter can be kept constant (immediate shear), or allowed to escape (slow shear). It is possible also in this apparatus to observe the change in voids ratio which usually occurs during the shearing of sandy soils, as well as the pore-water pressure. Furthermore, by relatively simple adaptations of the apparatus, it is possible to bring about a preconsolidation of the sample before it is subjected to compression and failure by shear.

Certain techniques with the triaxial compression machine allow the determination from one sample of three or more Mohr circles (as previously mentioned on page 25), which affords considerable saving in testing time and the number of samples necessary for establishing the cohesion and angle of internal friction of a soil. Lastly, the samples being of 1½ inch diameter, they are easier to obtain in the field and require less preparation in the laboratory than shear box samples.

CHAPTER III

STABILITY OF EARTH SLOPES

Sand Slopes

In the design of safe slopes for sandy soils, the angle i made by the slope with the horizontal should be smaller than the angle of internal friction of the sand, ϕ.

Normally in loose sands the angle of friction is about 32°, but this angle increases to 40° with very dense sands. It is important to remember that the angle of slope for stability of a cohesionless soil is independent of the height, which may be indefinite. Furthermore, the weight of the material does not affect the stability of the slope, therefore the safe angle for a submerged sand slope is the same as that for one composed of dry sand, with the exception of the special case of damp sand, which has a high angle of repose due to capillary attraction.

Special conditions exist with partially submerged sand slopes affected by tidal conditions which may cause the stability of a fine sand slope to be considerably less than that of dry sand. Assuming the angle of the safe slope with the horizontal is i then for submerged cohesionless soil slopes

$$\text{Tan i} = \frac{\rho - 1}{\rho + e} \tan \phi \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Conditions which exist with submerged slopes subject to "sudden draw-down" may be caused in a similar way with embankments of fine sand exposed to rainstorms sufficiently heavy to result in saturation of the sand fill.

Problem 29. A sample of sand is found to have an angle of internal friction of 32°, a porosity of 35 per cent. and an absolute density of 2·7. What would be the maximum slope to which this material could be tipped if it will be subjected to tidal conditions?

From equation (3),

$$\text{Voids ratio, } e = \frac{n}{1 - n} = \frac{0{\cdot}35}{1 - 0{\cdot}35} = 0{\cdot}54$$

From equation (19),

$$\text{Tan i} = \frac{\rho - 1}{\rho + e} \tan \phi = \frac{2{\cdot}7 - 1}{2{\cdot}7 + 0{\cdot}54} \times 0{\cdot}6249 = 0{\cdot}3279$$

Therefore, maximum angle of slope for stability, i = 18°.

It will be observed from this problem that although the maximum angle of slope for dry sand is 32°, the submerged slope of the same material would be unstable if tipped to a greater angle than 18°.

Clay Slopes

A slip which has taken place in a clay slope has three definite characteristics : a crack appears at the top of the bank, a portion of the material in the bank slips downward and there is a heave at the toe, as indicated in Fig. 11. In a bank of homogeneous

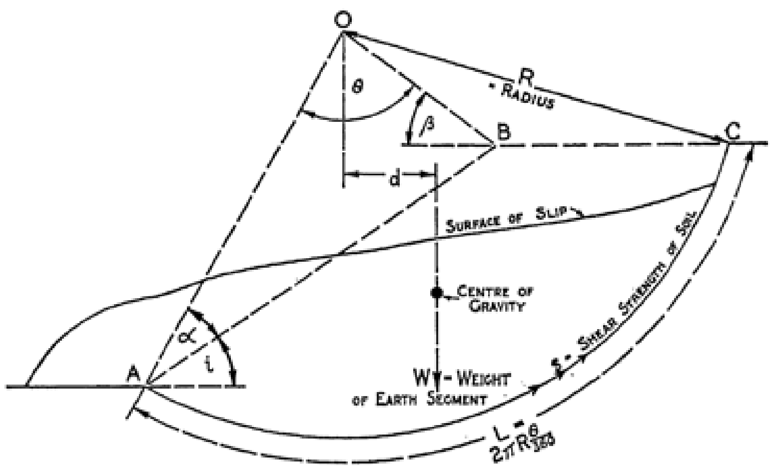


FIG. 11.—Stability of a Clay Slope.

clay material the slip line of failure in the slope closely follows the arc of a circle, and for stability.

$$W \times d = L \times s \times R \quad . \quad . \quad . \quad . \quad (20)$$

where W = weight of the segment of soil of unit thickness,
L = length of arc segment, AC,
R = radius of the cylindrical surface of shear,
d = distance of the centre of gravity of the segment from a vertical through the centre of curvature, and
s = average intensity of shear resistance per unit area of the cylindrical surface.

From the above formula it will be observed that in order to investigate the stability of a clay slope it is necessary to ascertain the weight of the soil, the apparent cohesion and the angle of internal

friction of the soil concerned. These data may be obtained from shear tests as described in Chapter II under the heading of "Shear Strength."

It must be emphasised that whilst the height of sand slopes do not in any way affect the safe angle of slope, clay slopes are entirely different and the safe slope is a function of the height. Sands possess an angle of repose, whilst clays do not have such a characteristic, but their behaviour is measured by their shear strength.

If the factor of safety of a clay slope is F, then

$$F = \frac{L \times s \times R}{W \times d} \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

The factor of safety cannot be considered as something absolute, as slopes with factors of safety less than unity have proved to be stable, but in the design of new works for cuttings and embankments it is advisable to maintain a safety factor between 1·25 and 1·5.

The centre of the critical circle is found by trial and error for the minimum value of the factor of safety, and the following notes will assist in the determination of the centre for this circle :—

(1) If the shear strength increases with depth, or if the slopes are steeper than 45°, then use the following table for values of angles α and β to find the centre O as indicated in Fig. 11.

TABLE 4.

Fellenius's Construction for Centre of Rotation.

Slope.	Angle of slope.	Angle α.	Angle β.
1 in 0·58	60°	29°	40°
1 in 1	45°	28°	37°
1 in 1·5	33° 47′	26°	35°
1 in 2	26° 34′	25°	35°
1 in 3	18° 26′	25°	35°
1 in 5	11° 19′	25°	37°

(2) If slopes are flatter than 45°, or if the clay is homogeneous, then the centre of the critical circle lies on a vertical through the mid-point of the slope. The circle tends to be deep and would tangent at an underlying layer of harder clay if such a stratum exists.

(3) When making adjustments for the centre of the critical circle, horizontal movements are more sensitive than vertical movements.

The following problems will indicate the methods employed in investigating or designing clay slopes.

Problem 30. A railway cutting is to be made in a clay which tends to increase in shear strength with depth. The soil has a weight of 120 lb. per cu. ft. and an average shear resistance of 600 lb. per sq. ft. (it is to be assumed that the angle of internal friction is nil). The depth of the cutting is 26·5 ft., and it is proposed to adopt slopes of 1 to 2. Using the values given in Table 4, ascertain the centre of the critical circle and calculate the factor of safety for the proposed slopes.

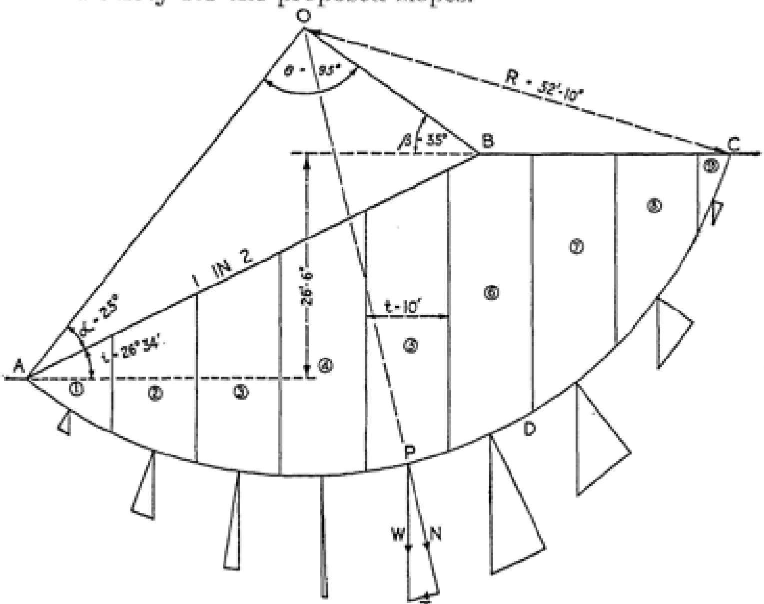


FIG. 12.—Method of Slices for Clay Slopes. (Problem 30.)

With reference to Fig. 12, angles α and β equal 25° and 35° (obtained from Table 4) are constructed to give the centre O, and with this centre the arc AC is drawn. The figure ABCD is divided into slices of equal width t, which in this case is 10 ft. The weight of each slice is proportional to the centre ordinate, and the average height of each slice is plotted vertically below P on the circle at these centre ordinates as W. This value for W is resolved into its normal and tangential components, N and T, by drawing OP produced for the normal component, and the tangential component at right angles to T from the length W.

The values of N and T for all the slices are added together and multiplied by the width, t (= 10 ft.), and by the weight of

the soil, w (= 120 lb. per cu. ft.), assuming that the whole slope has unit thickness.

$$\text{The factor of safety,}\quad F = \frac{Lc + \tan \phi \Sigma N}{\Sigma T} \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

From Fig. 12 the following table can be drawn up :—

Slice No.	Normal component, N.	Tangential component, T.	Other data.
1	5·2	− 3·1	
2	15·3	− 5·2	$L = 2\pi \times R \times \frac{\theta}{360}$.
3	23·5	− 3·0	
4	28·6	+ 1·9	$= 2 \times 3{\cdot}142 \times 52{\cdot}83$
5	31·3	+ 7·9	$\times\ 120 \div 360$
6	30·3	+ 13·9	= 110 ft.
7	21·3	+ 16·9	c = 600 lb./sq. ft.
8	10·3	+ 13·9	ϕ = zero.
9	2·0 × ½	+ 5·2 × ½	w = 120 lb./cu. ft.
	166·5 × 120 × 10	44·9 × 120 × 10	t = 10 ft.
	199,800 lb.	53,880 lb.	

$$\text{As } \phi \text{ is equal to zero,}\quad F = \frac{Lc}{\Sigma T} = \frac{110 \times 600}{53880} = 1{\cdot}2$$

It may be assumed from this result that the cutting would be stable with a factor of safety of 1·2, although it would be desirable to increase this to 1·3 or 1·4 if practicable.

(*Note.*—This problem assumes that the centre found from Table 4 is the centre of the critical circle, but it is necessary to investigate other trial centres until the critical circle is determined, as will be shown in the following problem.)

Problem 31. A cutting for a roadway is to be excavated in a clay which has the following properties :—

Weight of soil, 120 lb. per cu. ft.
Apparent cohesion, 800 lb. per sq. ft.
Angle of internal friction, 6 degrees.

The cutting is to be 30 ft. deep, and slopes of 1 to 2 are proposed. Ascertain the critical circle of failure and determine the factor of safety.

With reference to Fig. 13, the first centre, O_1, is obtained by assuming that ϕ equals zero and using Table 4 for obtaining values of α and β. For a slope of 1 in 2 the angles are $\alpha = 25°$ and $\beta = 35°$. The circle with centre O_1 and the radius R_1 ($= O_1A$) is drawn and the segment divided into slices of equal

width of 10 ft. Triangles of forces for W, N and T are drawn for each slice as described in the previous problem, No. 30. The data is tabulated as indicated on page 37.

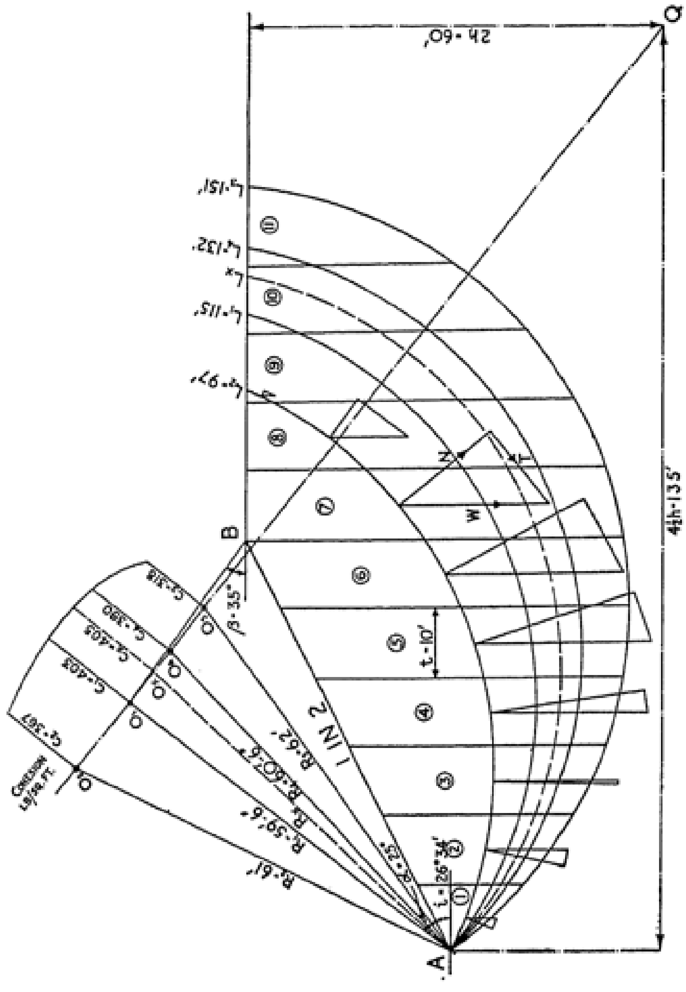


FIG. 13.—The Fellenius Construction. (Problem 31.)

Taking moments about O_1 for equilibrium results in the following equation :—

$$\text{Cohesion per unit area, } c = \frac{\Sigma T - \tan\phi \Sigma N}{L} \quad . \quad . \quad . \quad . \quad (23)$$

$$c_1 = \frac{(60{\cdot}9 \times 1200) - 0{\cdot}1051\,(211{\cdot}8 \times 1200)}{115} = 403 \text{ lb./sq. ft.}$$

A point Q is now ascertained as shown in Fig. 13 by taking a distance of 4·5h from the toe at a depth of 2h below the top of the slope. A line is drawn from Q to O_1 and produced. Along this line adjacent to O_1 other centres for trial circles are taken at O_2, O_3 and O_4, with radii of R_2 ($= O_2A$) $= 61$ ft., R_3 ($= O_3A$) $= 62$ ft. and R_4 ($= O_4A$) $= 60·5$ ft., and lengths of arcs, $L_2 = 97$ ft., $L_3 = 151$ ft. and $L_4 = 132$ ft. respectively.

Centre No.	O_1.		O_2.		O_3.		O_4.	
Arc length.	$L_1 = 115'$.		$L_2 = 97'$.		$L_3 = 151'$.		$L_4 = 132'$.	
Radius.	$R_1 = 59' 6''$.		$R_2 = 61'$.		$R_3 = 62'$.		$R_1 = 60' 6''$.	
Slice No.	Normal.	Tangential.	N.	T.	N.	T.	N.	T.
1	5·0	− 3·0	4·2	− 1·7	6·6	− 5·4	4·4	− 4·2
2	15·2	− 5·65	11·8	− 2·2	18·5	−12·7	14·7	− 9·0
3	23·5	− 4·5	18·0	− 0·5	30·4	−13·3	25·2	− 8·6
4	30·4	− 0·5	22·5	3·5	40·0	−10·1	33·2	− 5·1
5	33·8	5·5	24·2	8·0	48·0	1·7	39·2	1·3
6	35·0	12·0	23·0	12·5	53·0	4·8	42·2	9·4
7	30·7	17·0	17·0	14·0	51·4	17·0	40·1	16·3
8	29·0	24·8	7·1	9·3	45·0	23·0	32·1	21·3
9	10·0	14·0	$\left.\begin{matrix}0·8\\ \times\ 0·2\end{matrix}\right\}$	$\left.\begin{matrix}1·5\\ \times\ 0·2\end{matrix}\right\}$	36·5	27·0	21·5	22·2
10	$\left.\begin{matrix}1·8\\ \times\ 0·3\end{matrix}\right\}$	$\left.\begin{matrix}4·0\\ \times\ 0·3\end{matrix}\right\}$	—	—	24·7	26·7	9·7	17·2
11	—	—	—	—	10·6	19·6	$\left.\begin{matrix}2·7\\ \times 0·25\end{matrix}\right\}$	$\left.\begin{matrix}5·7\\ \times\ 0·25\end{matrix}\right\}$
Totals	211·8	60·9	128·0	43·1	364·7	78·3	263·0	62·2

The method of slices is repeated for each circle, and from equation (23) the following results are obtained :—

$$c_1 = \frac{(60·9 \times 1200) - 0·1051\,(211·8 \times 1200)}{115} = 403 \text{ lb./sq. ft.}$$

$$c_2 = \frac{(43·1 \times 1200) - 0·1051\,(128 \times 1200)}{97} = 367 \text{ lb./sq. ft.}$$

$$c_3 = \frac{(78·3 \times 1200) - 0·1051\,(364·7 \times 1200)}{151} = 318 \text{ lb./sq. ft.}$$

$$c_4 = \frac{(62·2 \times 1200) - 0·1051\,(263 \times 1200)}{132} = 390 \text{ lb./sq. ft.}$$

These values are plotted as indicated on Fig. 13, and the points joined to form a smooth curve. The crest of the curve giving the

maximum value for c_x and enabling the position of O_x to be determined, which is the centre of the critical circle. A re-calculation for this critical circle enables the factor of safety from equation (22) to be ascertained :—

$$F = \frac{(L \times c) + \tan \phi \Sigma N}{\Sigma T}$$

$$= \frac{(123 \times 800) + 0{\cdot}1051\,(237{\cdot}4 \times 1200)}{61{\cdot}6 \times 1200} = 2{\cdot}5$$

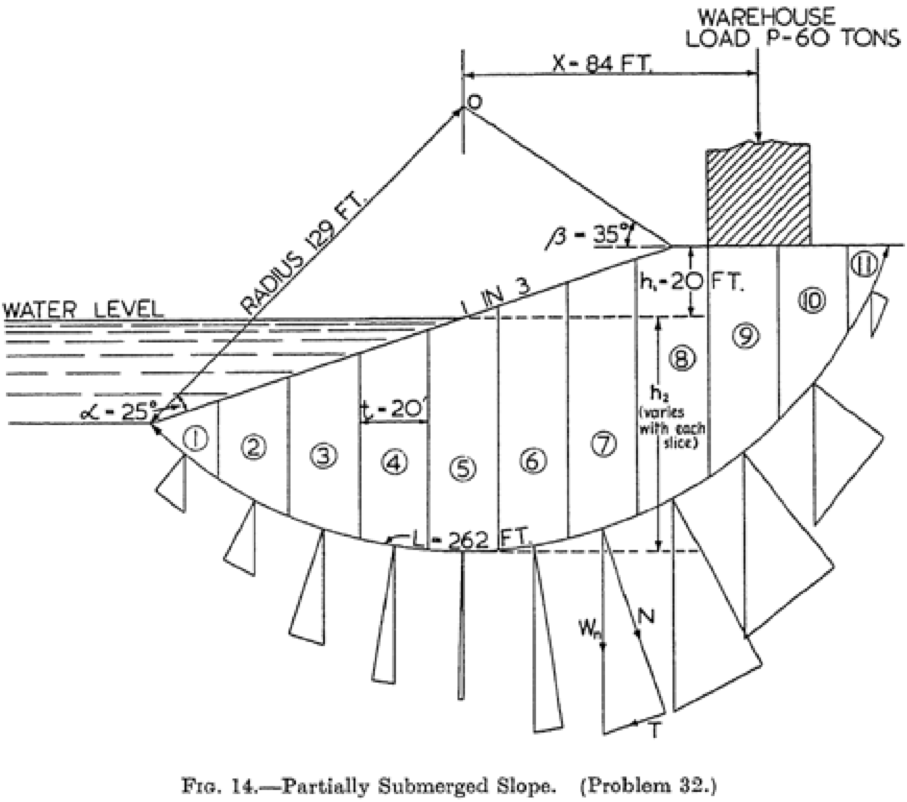


FIG. 14.—Partially Submerged Slope. (Problem 32.)

Problem 32. Investigate the stability of the partially submerged embankment shown in Fig. 14. The total height of the bank is 50 ft. and the submerged portion is 30 ft. At the top of the bank a warehouse is to be constructed at a centre line 25 ft. from the edge and exerting a load of 60 tons per lin. ft. along the bank. The soil is homogeneous clay with a cohesive strength of 700 lb. per sq. ft., specific gravity of 2·67 and voids ratio of 57

per cent. (Assume complete saturation below water-line and centre of critical circle can be obtained from Table 4.) Slope of bank 1 in 3.

With reference to Fig. 14, centre O is found by constructing angles $\alpha = 25°$ and $\beta = 35°$ as found from Table 4, and the critical circle is drawn. The segment is divided into slices of 20 ft. width. The vertical weight of each slice W_n is composed of two parts : the dry soil above the water line and the completely saturated soil below this line.

$$\therefore W_n = t(h_1w_1 + h_2w_2) \quad . \quad . \quad . \quad . \quad (24)$$

where h_1 denotes the distance from water level to ground level,
h_2 denotes the distance from water level or submerged ground level to the circular arc (this distance varies with each slice under consideration),
w_1 denotes the dry density of the soil calculated in the case of sands and porous fills from formula (6), but for clays where the soil is not completely dry, the saturated density from equation (7) is used, and
w_2 denotes the submerged density ascertained from equation (8).

Hence, the dry density

$$w_1 = \frac{\rho + e}{1 + e}\gamma_w = \frac{2{\cdot}67 + 0{\cdot}57}{1 + 0{\cdot}57} \times 62{\cdot}5 = 129 \text{ lb./cu. ft.}$$

and submerged density (from equation 8)

$$w_2 = \frac{\rho - 1}{1 + e}\gamma_w = \frac{2{\cdot}67 - 1}{1 + 0{\cdot}57} \times 62{\cdot}5 = 66{\cdot}5 \text{ lb./cu. ft.}$$

W_n is then resolved into the normal and tangential components, N and T, as described in the two previous problems.

The superload of the warehouse acts as a disturbing moment equal to Px, where x is the distance indicated in Fig. 14

The disturbing moment $= R\Sigma T + Px$

The restoring moment $= (Lc + \tan \phi\Sigma N)R$

Therefore the factor of safety $= \dfrac{(Lc + \tan \phi\Sigma N)R}{R\Sigma T + Px} \quad . \quad (25)$

Tabular results are given below :—

Slice No.	Normal component, N.	Tangential component, T.	Other data.
1	24,600 lb.	−20,500 lb.	L = 262 ft.
2	37,500 ,,	−20,500 ,,	R = 129 ,,
3	60,000 ,,	−20,000 ,,	x = 84 ,,
4	78,000 ,,	−13,600 ,,	P = 134,400 lb.
5	69,760 ,,	−1,600 ,,	c = 700 lb. per sq. ft.
6	104,000 ,,	14,500 ,,	t = 20 ft.
7	110,000 ,,	34,500 ,,	
8	109,000 ,,	55,500 ,,	
9	83,000 ,,	64,800 ,,	
10	51,300 ,,	62,000 ,,	
11	12,000 ,,	24,000 ,,	
Totals .	739,160 lb.	179,100 lb.	

Assuming the clay has no angle of internal friction, ϕ, the factor of safety (from equation (25) above)

$$= \frac{Lc \times R}{R\Sigma T + Px} = \frac{262 \times 700 \times 129}{(129 \times 179{,}100) + (84 \times 134{,}400)} = 0{\cdot}69$$

Factor of safety without load of warehouse

$$\frac{Lc \times R}{R\Sigma T} = \frac{262 \times 700 \times 129}{129 \times 179{,}100} = 1{\cdot}03$$

It will be realised from these results that the partially submerged slope is just stable without the load due to the warehouse. Such a building reduces the factor of safety to 0·69, and would therefore cause instability. Piles driven to a depth well below the critical slip circle would be necessary to support such a structure.

(*Note.*—A clay soil with a cohesive strength of 700 lb. per sq. ft. and an angle of internal friction of 18½° would give a factor of safety of 1·5 for such conditions, allowing for the superload due to the warehouse.)

Problem 33. Investigate the failure of the cutting slope shown in Fig. 15. The cutting is 35 ft. deep in a clay with a cohesive strength of 500 lb. per sq. ft. and weight of 120 lb. per cu. ft., and slope of 1 in 2½. At a depth of 40 ft. below the top of the slope a layer of soft clay exists 3 ft. thick, with a cohesive strength of 300 lb. per sq. ft. and weight of 100 lb. per cu. ft.

The centre of the critical circle of failure lies on the vertical through the top of the slope, and the radius of the circle is such that the circle tangents just below the upper surface of the soft

clay layer, as shown in Fig. 15. The line of failure then follows the soft clay to the foot of the slope.

The method of slices as previously described is introduced, and it is assumed that the weight of the segment of soil tending to slip downwards acts at about the third of the height of the cutting plus the depth to the soft clay layer.

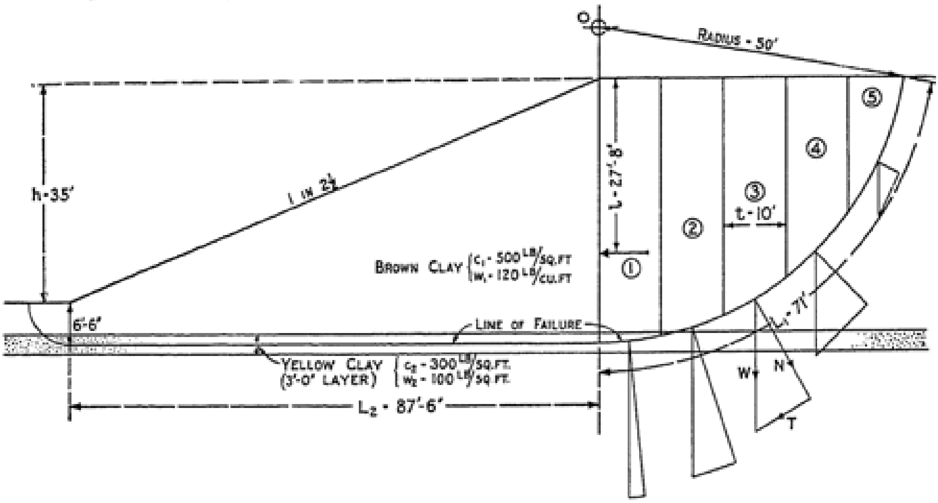


FIG. 15.—Slope with Varying Shear Strength. (Problem 33.)

Tabular results of the method of slices are as follows :—

Slice No.	Normal component, N.	Tangential component, T.	Other data.
1	49,500 lb.	5,000 lb.	L = 71 ft.
2	45,000 ,,	14,500 ,,	R = 50 ,,
3	36,500 ,,	21,000 ,,	c_1 = 500 lb. per sq. ft.
4	23,800 ,,	22,800 ,,	w_1 = 120 lb. per cu. ft.
5	5,000 ,,	7,500 ,,	
Totals .	159,800 lb.	70,800 lb.	

The disturbing moment $= R\Sigma T$.

The restoring moment $=$ shear resistance along arc + shear resistance of soft clay layer acting at third of height.

$$= R(L_1c_1 + \tan\phi\Sigma N) + L_2c_2l.$$

Therefore, the factor of safety

$$= \frac{R(L_1c_1 + \tan\phi\Sigma N) + L_2c_2l}{R\Sigma T} \quad . \quad . \quad . \quad (26)$$

If ϕ is equal to zero, the factor of safety

$$= \frac{RL_1c_1 + L_2c_2l}{R\Sigma T}$$

$$= \frac{(50 \times 71 \times 500) + (87\cdot5 \times 300 \times 27\cdot67)}{50 \times 70{,}800} = 0\cdot71$$

From the above result it is apparent that the cutting is unstable, and with a safety factor of 0·71 a failure of the slope by the occurrence of a slip is to be expected. A flatter cutting slope and the provision of a berm at half the height of the slope would enable a stable construction to be attained.

Taylor's Curves for Analysis of Clay Slopes

The method of slices used in investigating the previous problems was developed by Fellenius, and is most useful for considering slopes composed of clay layers of differing shear strengths, partially submerged slopes or slopes of uneven section. However, for the investigation of the stability of homogeneous clay cuttings or slopes of regular section, a method has been developed by Taylor in which a " stability number " is obtained from a series of graphs which enables the factor of safety to be rapidly calculated.

The " stability number," $N = \dfrac{c}{whF}$ . . . . (27)

where c = cohesion per unit area,
w = weight of soil per unit volume,
h = height of slope,
F = factor of safety.

In the curves given in Fig. 16 the stability number is plotted vertically, and by appropriate reference to the curves for angles of internal friction, ϕ for values between 0° and 25°, the safe angle of slope can be determined. Alternatively, when considering an existing slope, the existing angle of repose of the soil will enable the stability number to be obtained from the curves, and using formula (27) the factor of safety can be computed.

The curves may be used for calculating the stability of submerged slopes by using a " weighted " angle of internal friction, ϕ_w.

" Weighted " angle of internal friction,

$$\left.\begin{aligned} \phi_w &= \frac{\text{Submerged weight}}{\text{Weight in air}} \times \phi \\ &= \frac{(\rho - 1)}{(\rho + e)} \times \phi \end{aligned}\right\} \quad . \quad . \quad . \quad (28)$$

The second series of curves given in Fig. 17 is an extension of the general case when ϕ equals zero and there is a stratum of underlying rock or hard material at a depth Dh below the top of the bank. The circle of failure does not penetrate the hard material, and D is known as the depth "factor."

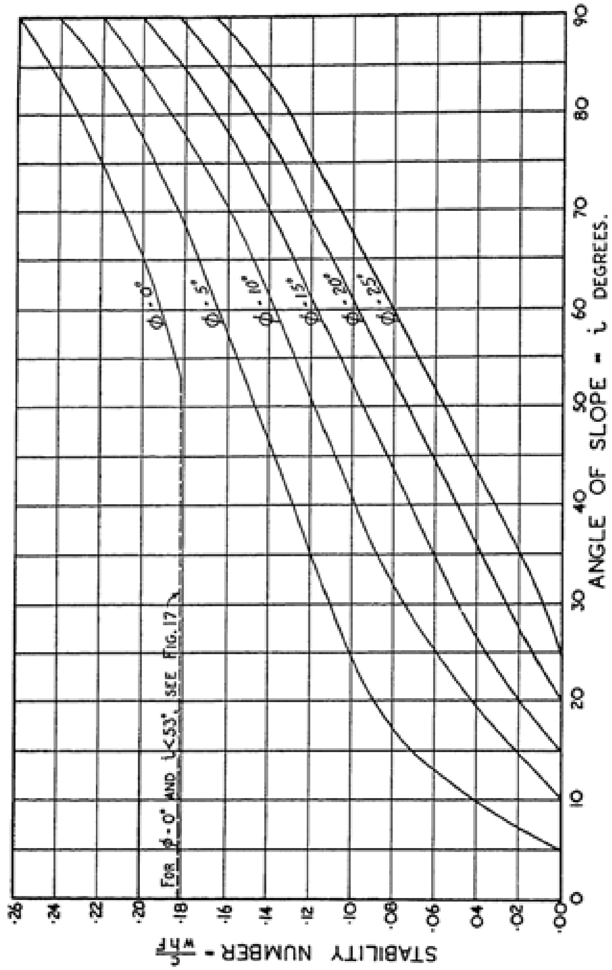


Fig. 16.—Taylor's Curves.

Two conditions exist, (1) where the circle of failure passes below the toe of the slope, and (2) where the circle cannot do so. In the former case the curves give a value for n which enables the distance nh, the slip circle cuts the horizontal in front of the toe to be calculated.

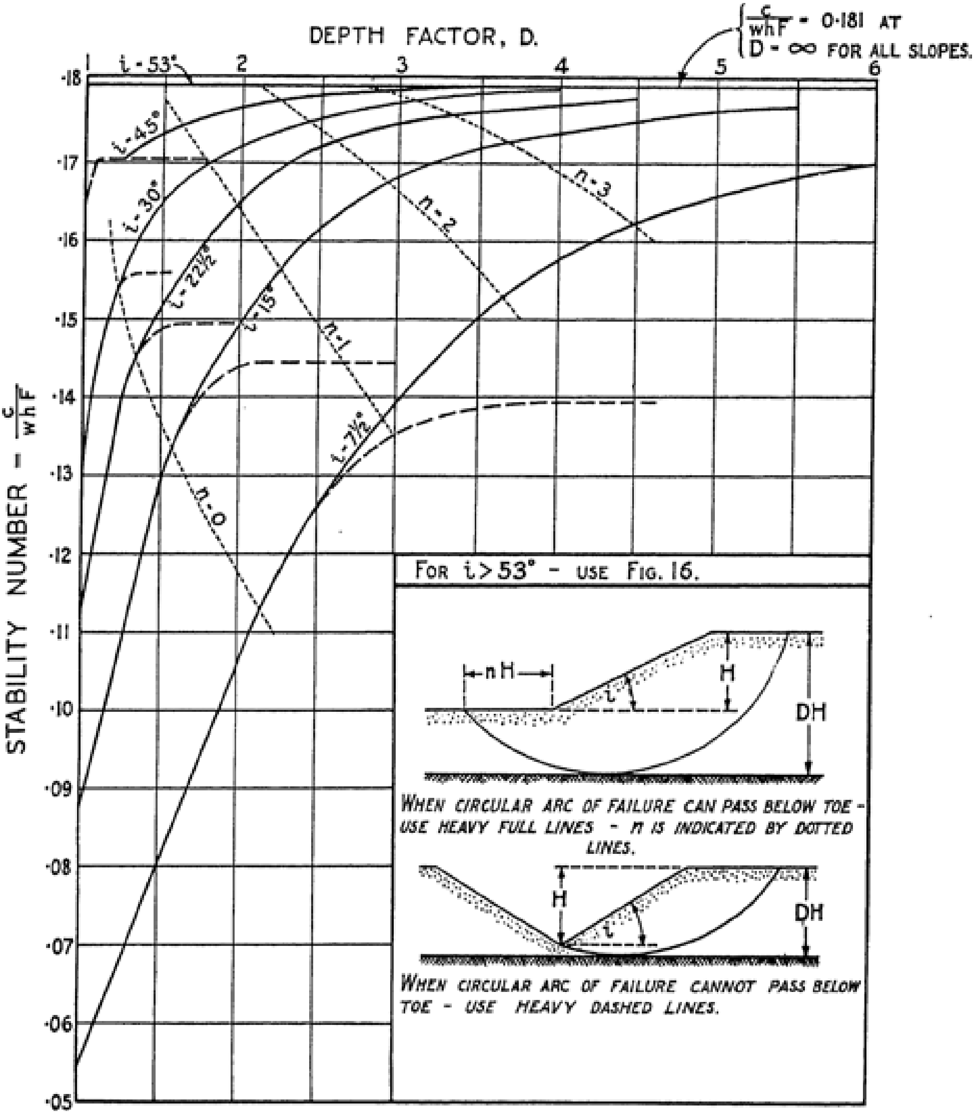


Fig. 17.—Taylor's Curves.—Effect of Depth Limitation.

Problem 34. A cutting 30 ft. deep for a main road is to be constructed through a clay soil which has a cohesion of 400 lb. per sq. ft. and an angle of internal friction of 16°, as determined by shear-box tests. The weight of the clay is 118 lb. per cu. ft., and it is proposed to adopt slopes of 1 to $1\frac{1}{2}$. What is the safety factor obtained from Taylor's curves?

For an angle of slope, $i = 33° \ 47'$ (= 1 to $1\frac{1}{2}$) and $\phi = 16°$, the "stability number"

$$= 0{\cdot}068$$

$$= \frac{c}{whF}$$

∴ factor of safety

$$= \frac{400}{0{\cdot}068 \times 118 \times 30} = 1{\cdot}66$$

A factor of safety of 1·66 is ample for stability for this type of cutting.

Problem 35. A trench excavation is to be made in a clay soil with a cohesion of 350 lb. per sq. ft. and an angle of internal friction of 5°. What is the depth to which such a trench can be taken without timbering, allowing a factor of safety of 1·1 for such temporary work? The weight of the soil is 115 lb. per cu. ft.

For an angle of slope, $i = 90°$ and $\phi = 5°$, the "stability number"

$$= 0{\cdot}24$$

$$= \frac{c}{whF} = \frac{350}{1{\cdot}1 \times 115 \times h}$$

∴ Depth of excavation unsupported

$$= h = \frac{350}{1{\cdot}1 \times 115 \times 0{\cdot}24} = 11{\cdot}5 \text{ ft.}$$

Problem 36. A new cut for a canal 16 ft. deep is being made in a sandy clay with a cohesion of 150 lb. per sq. ft. and an angle of internal friction of 12°. The voids ratio of the soil is 0·9 and the specific gravity 2·6. If the banks are constructed with slopes of 1 to 1, what is the factor of safety when the canal is completely full? Would the canal banks be stable if a "sudden draw-down" of the water occurred?

From equation (8), the submerged density of the soil

$$\frac{\rho - 1}{1 + e}\gamma_w = \frac{(2{\cdot}6 - 1)}{(1 + 0{\cdot}9)} \times 62{\cdot}5 = 52{\cdot}6 \text{ lb./cu. ft.}$$

For an angle of slope, $i = 45°$ and $\phi = 12°$, the "stability number" $= 0{\cdot}10$.

From equation (27),

$$F = \frac{c}{Nwh} = \frac{150}{0{\cdot}1 \times 52{\cdot}6 \times 16} = 1{\cdot}78$$

The factor of safety of 1·78 for the canal banks when the canal is full can be regarded as ample and satisfactory.

For "sudden draw-down" conditions, the "weighted" angle of internal friction as obtained from equation (28)

$$= \phi_w = \frac{(\rho - 1)}{(\rho + e)} \times \phi = \frac{(2{\cdot}6 - 1)}{(2{\cdot}6 + 0{\cdot}9)} \times 12° = 5{\cdot}5°$$

For an angle of slope, $i = 45°$ and $\phi = 5{\cdot}5°$, the "stability number" $= 0{\cdot}138$.

From equation (27),

$$F = \frac{150}{0{\cdot}138 \times 52{\cdot}6 \times 16} = 1{\cdot}29$$

The factor of safety is reduced to 1·29 when "sudden draw-down" conditions occur, and this may be regarded as satisfactory.

Problem 37. A cutting 20 ft. deep is being constructed through a soft clay with a cohesive resistance of 500 lb. per sq. ft. which overlies a Lower Greensand stratum at a depth of 35 ft. of high shear strength of approximately 3,000 lb. per sq. ft. The soft clay has a weight of 120 lb. per cu. ft. It may be assumed that the angle of internal friction is very small and may be neglected. Find the angle of slope necessary to provide a factor of safety of 1·3.

With reference to Fig. 17, the "depth factor," $D = 1{\cdot}75$.

By equation (27) the "stability number"

$$= \frac{c}{whF}$$

$$= \frac{500}{1{\cdot}3 \times 20 \times 120} = 0{\cdot}16$$

By the use of Taylor's curves, angle of slope, $i = 22{\cdot}5°$ and $n = 0{\cdot}7$, thus the slip circle cuts the surface in front of the toe of the slope a distance of $nh = 0{\cdot}7 \times 20 = 14$ ft.

(In Fig. 11 on page 35, and Fig. 12 on page 37, the angle θ should denote angle AOC.)

CHAPTER IV

STABILITY OF EARTH SLOPES IN CUTTINGS—DESIGN

In the previous chapter reference was made to the use of Taylor's graphs for making a quick analysis of the stability of a clay slope. The design of slopes for cuttings in clay soils may be easily carried out by referring to the graph shown in Fig. 18, provided the shear

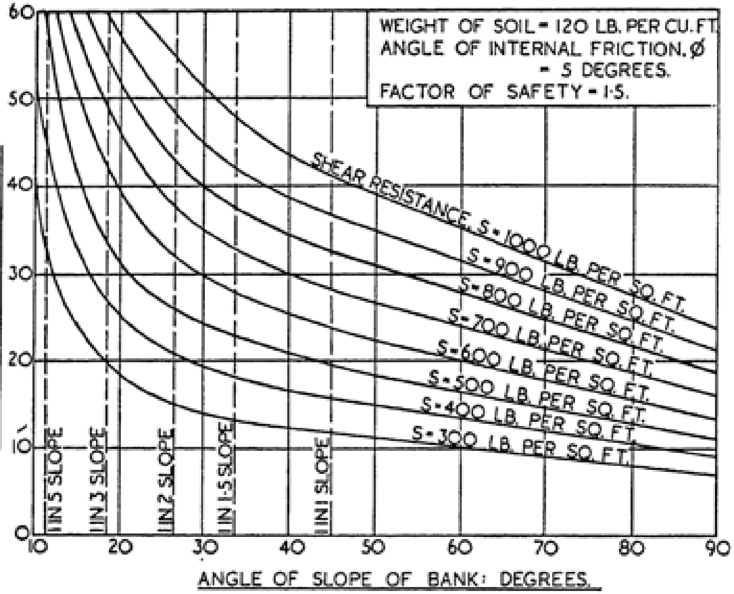


FIG. 18.—Stability of Earth Slopes.

strength of the soil is known. This particular graph allows for a factor of safety of 1·5 with soils having an angle of internal friction of 5° and weight of 120 lb. per cu. ft., but other graphs may be developed in a similar manner from Taylor's curves.

Problem 38. A cutting of the type shown in Fig. 19 is to be excavated through clay with an average shear strength of 600 lb. per sq. ft., the angle of internal friction is zero and the weight is 120 lb. per cu. ft. The maximum depth of the cutting is 20 ft.,

but boring records indicate an underlying stratum of rock at a depth of 35 ft.

(i) What slope should be adopted to give a factor of safety of 1·5? (Use Fig. 17.)

(ii) If the ground level to one side of the centre line is to be excavated to the full depth of the cut, what alteration will be required to the slope to maintain the factor of safety of 1·5?

(i) From the data given and using the equation (27) :—

$$N = \frac{c}{Fwh} = \frac{600}{1{\cdot}5 \times 120 \times 20} = 0{\cdot}17$$

Also, $Dh = 35$ ft. or $D = 1{\cdot}75$.

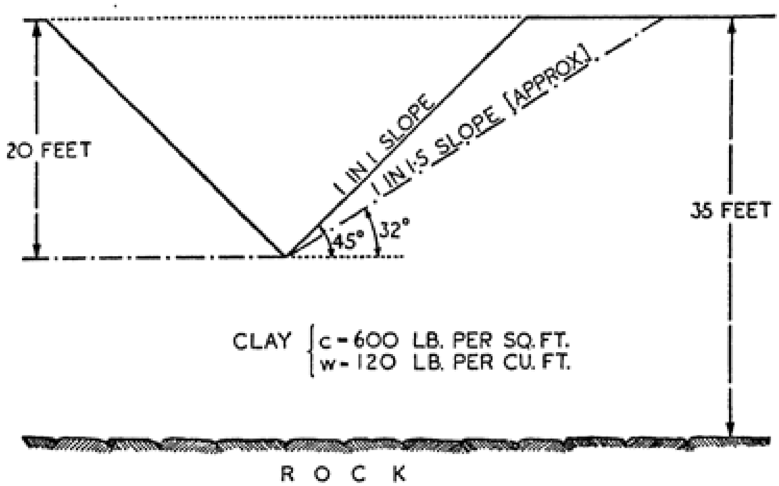


Fig. 19.—Cutting in Clay. (Problem 38.)

With reference to Fig. 17, for $N = 0{\cdot}17$ and $D = 1{\cdot}75$, read $i = 45°$, as the slip circle cannot pass below the toe of the slope (indicated by dashed lines on graph).

Therefore, the design slope should be 45° or 1 to 1 batter.

(ii) In the second case the slip circle can pass under the toe of the slope, and reading the full line on Fig. 17 for $N = 0{\cdot}17$ and $D = 1{\cdot}75$, $i = 32°$, or slightly flatter than a $1\frac{1}{2}$ to 1 slope.

This problem clearly indicates the difference in design for these two conditions of cutting slopes.

Problem 39. A railway cutting 35 ft. deep is to be made through a clay soil which has a soft upper layer 15 ft. deep of shear strength 350 lb. per sq. ft. The underlying clay has a shear strength of 790 lb. per sq. ft., and both clays have an angle of internal friction of 5° with a weight of 120 lb. per cu. ft. Design

a suitable compound slope for the cutting to ensure a factor of safety of 1·5.

Referring to Fig. 18, the safe slope for a cutting 35 ft. deep in clay of shear strength 790 lb. per sq. ft. is 35°, or approximately a $1\frac{1}{2}$ to 1 batter. The lower portion of the slope up to a height of 20 ft. should be designed with a $1\frac{1}{2}$ to 1 slope, and it may be estimated that the upper portion should be reduced to a 3 to 1 slope (*i.e.*, the safe slope for a clay of shear strength 350 lb. per sq. ft. for a height of about 25 ft.).

Fig. 20 indicates the compound slope, and a circular arc analysis is then taken out, the result of which is given below.

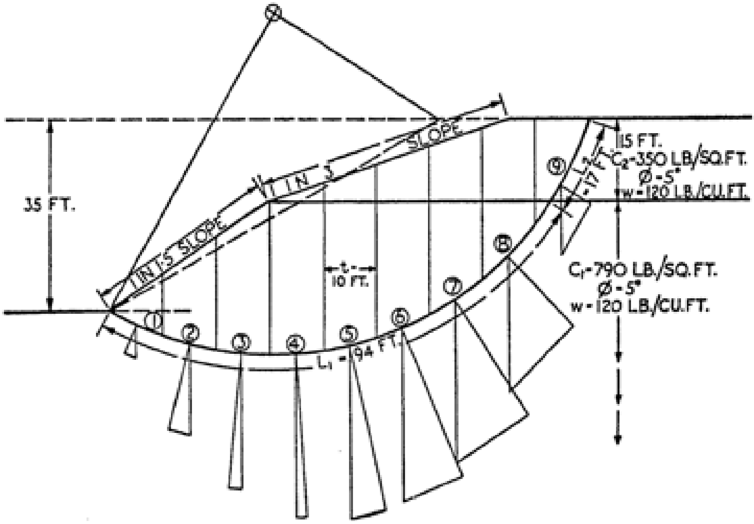


FIG. 20.—Compound Cutting Slope. (Problem 39.)

Slice No.	Normal component, N.	Tangential component, T.	Other data.
1	5·5	− 2·5	$L_1 = 94$ ft., $L_2 = 17$ ft. $= 111$ ft.
2	16·0	− 4·0	
3	24·8	− 2·3	$c_1 = 790$ lb. per sq. ft.
4	30·0	2·0	$c_2 = 350$ lb. per sq. ft.
5	31·0	7·0	$\phi = 5$ degrees.
6	29·5	12·0	w = 120 lb. per cu. ft.
7	25·0	15·7	t = 10 ft.
8	18·0	17·5	
9	6·5	10·5	
	186·3 × 10 × 120 = 223,560 lb.	55·9 × 10 × 120 = 67,080 lb.	

From equation (22)

$$F = \frac{(L_1 \times c_1) + (L_2 \times c_2) + \tan \phi \Sigma N}{\Sigma T}$$

$$= \frac{(94 \times 790) + (17 \times 350) + (0{\cdot}0875 \times 223{,}560)}{67{,}080}$$

$$= \frac{74{,}260 + 5{,}950 + 19{,}560}{67{,}080}$$

$$= 1{\cdot}5$$

Should the factor of safety vary from the desired value of 1·5, then an adjustment should be made in the 3 to 1 slope so that the requisite factor is obtained.

Problem 40. A railway cutting in clay exists in the form indicated in Fig. 21. The depth of the cutting is 28 ft., and during the original construction the excavation was tipped along the top

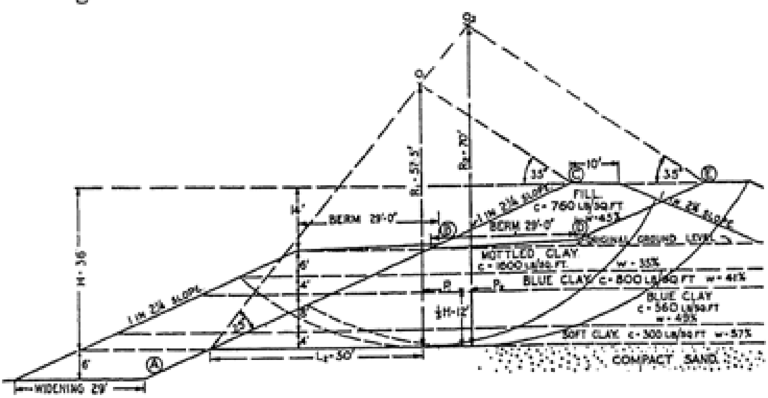


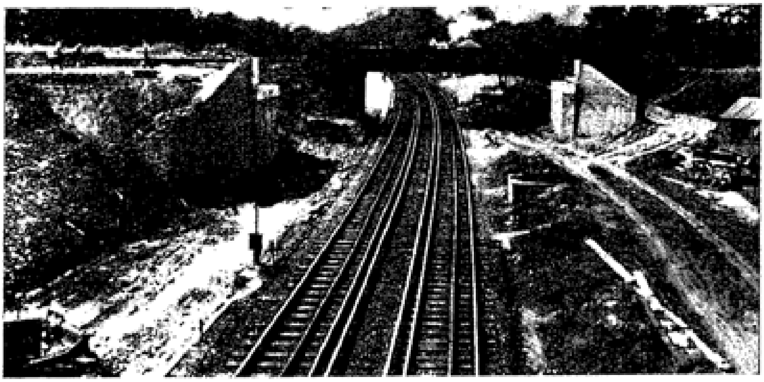
FIG. 21.—Widening of Cutting. (Problem 40.)

of each slope to form flood-banks, but a horizontal benching or berm 29 ft. wide was provided at the original ground level.

In order to provide additional permanent way, it is desired to widen the cutting on one side by 29 ft. Fig 21 gives the necessary information relating to soil conditions as found by borings. Design suitable cutting slopes for the widening, adopting a factor of safety of 1·25.

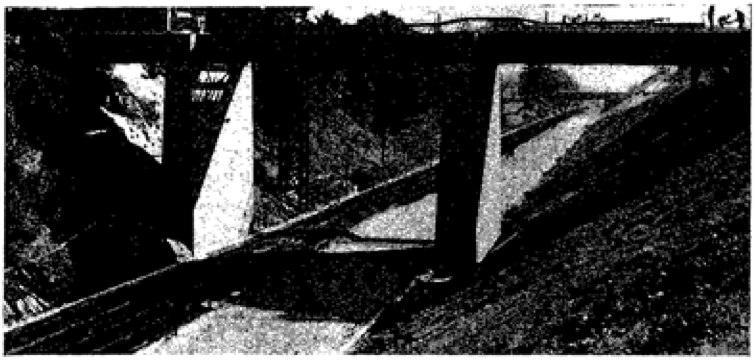
From examination of the data in Fig. 21, it is reasonable to assume that the circular arc of failure in this cutting will be of the type referred to in Problem 33 and the arc will tangent at the upper horizon of the compact sand. The blue clay in contact with this sand is of low shear strength—300 lb. per sq. ft.—and the line of failure will closely follow this strata horizontally.

(a)



(b)

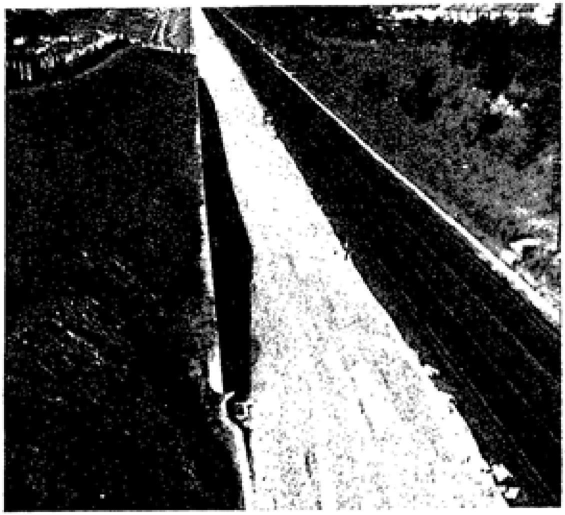
(c)



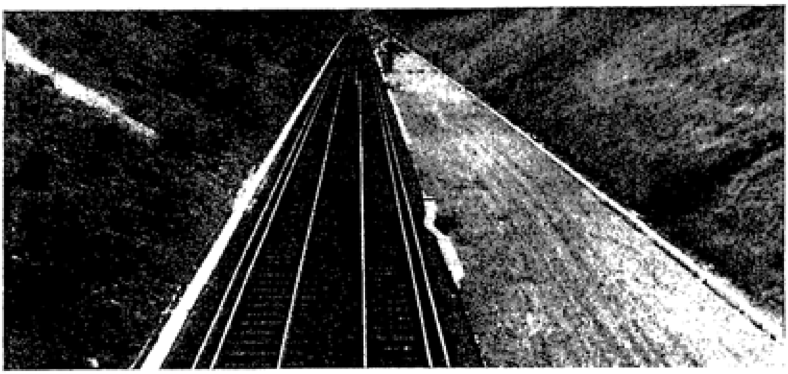
[*Photos 23(a) to 23(f) by courtesy of British Railways, Southern Region.*

Railway Widening Works between Bickley Junc. and Swanley Junc., 1958/1959.

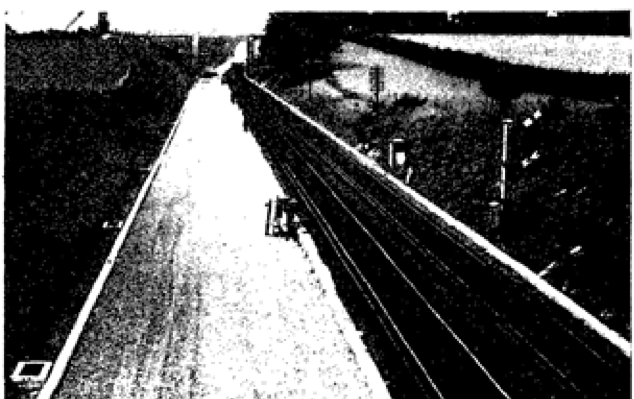
FIGS. 21 (*a*).—Bridge 154A looking west.
21 (*b*).—View looking towards Bickley Junction.
21 (*c*).—Bridge 92 London side.



(d)



(e)



(f)

Figs. 21 (*d*).—View from Bridge 92 looking towards St. Mary Cray.
21 (*e*).—From Bridge 93 looking towards Swanley.
21 (*f*).—From Bridge: looking towards Bridge 92.

First, the possibility of carrying out the widening could be undertaken by removing the soil in the lower portion of the slope and providing a continuous batter of $2\frac{1}{4}$ to 1 from A to B and C. The stability of this condition will now be investigated. In the manner previously described, the centre O_1 is ascertained and the following table drawn up.

Slice No.	Normal component, N.	Tangential component, T.	Other data.
1	24·5	2·5	R = 57·5 ft. t = 10 ft. w = 120 lb. per cu. ft. L_1 = 63 ft. c_1 = 800 lb. per sq. ft. L_2 = 50 ft. c_2 = 300 lb. per sq. ft. l = 28 ft. ϕ = zero.
2	28·0	8·2	
3	25·0	12·5	
4	19·5	15·3	
5	7·0	10·5	
× t × w = × 10 × 120	104·0 = 124,800 lb.	49·0 = 58,800 lb.	

Factor of safety,

$$F = \frac{R(L_1 \times c_1 + \tan \phi \Sigma N) + L_2 \times c_2 \times l}{R \times \Sigma T}$$

$$= \frac{57{\cdot}5 \times 63 \times 800 + 50 \times 300 \times 28}{57{\cdot}5 \times 58{,}800}$$

$$= 0{\cdot}98$$

It will be apparent that the proposal to widen the cutting in this manner will not be satisfactory, as the factor against failure is less than unity.

Slice No.	Normal component, N.	Tangential component, T.	Other data.
a	22·0	2·0	R = 70 ft. t = 10 ft. w = 120 lb. per cu. ft. L_1 = 73 ft. c_1 = 800 lb. per sq. ft. L_2 = 61 ft. c_2 = 300 lb. per sq. ft. l = 28 ft. ϕ = zero.
b	20·5	5·0	
c	19·0	7·0	
d	17·8	10·0	
e	13·5	11·5	
f	7·2	9·0	
× t × w = × 10 × 120	100·0 = 120,000 lb.	44·5 = 53,400 lb.	

Secondly, the possibility of widening the cutting and reproducing the berm as indicated by the profile ABDE will now be investigated. Conditions regarding the arc of failure will be similar, and after ascertaining the centre O_2 the results can be tabulated as in the preceding table (see p. 51).

Factor of safety,

$$F = \frac{R(L_1 + \tan\phi\Sigma N) + L_2 \times c_2 \times 1}{R\Sigma T}$$

$$= \frac{70 \times 73 \times 800 + 61 \times 300 \times 28}{70 \times 53{,}400}$$

$$= 1{\cdot}25$$

It will be realised from the above analyses that the widening to the cutting should be designed to incorporate a benching which can be provided at about 28 ft. above the railway. The benching should be provided with a fall towards the railway tracks, in order to assist drainage. The photographs in Figs. 22 and 23 illustrate the widening of a cutting under similar conditions to those referred to in this problem.

Toe Walls and Stone Pitching

The circular-arc method of analysis can be applied to investigate stability where it is proposed to adopt toe walls at the foot of a slope or to use stone pitching to face a slope.

The toe wall should be designed with consideration being given to the following points where clay or sandy clay soil exists :—

(*a*) The foundation should be taken down to a sufficient depth below the circular arc of failure, even if it is necessary to adopt sheet piling.

(*b*) An ample thickness of dry stone backing should be provided.

(*c*) Adequate arrangements are necessary to drain water away from the back of the wall to prevent percolation into the foundations, which would cause sliding due to increased moisture content.

Schemes involving widening of cuttings for roadways or railways may be economically undertaken by the provision of toe walls, as it is unnecessary to purchase additional land, and excavation, with the consequent danger due to the exposure of a fresh surface of soil to weathering agents, is avoided.

Stone pitching to a steeper angle than the stable angle of slope for the soil under consideration may be adopted for cutting widen-

FIG. 22.—Pilning Cutting. Construction of Berm for Widening of Cutting. (Problem 40.)

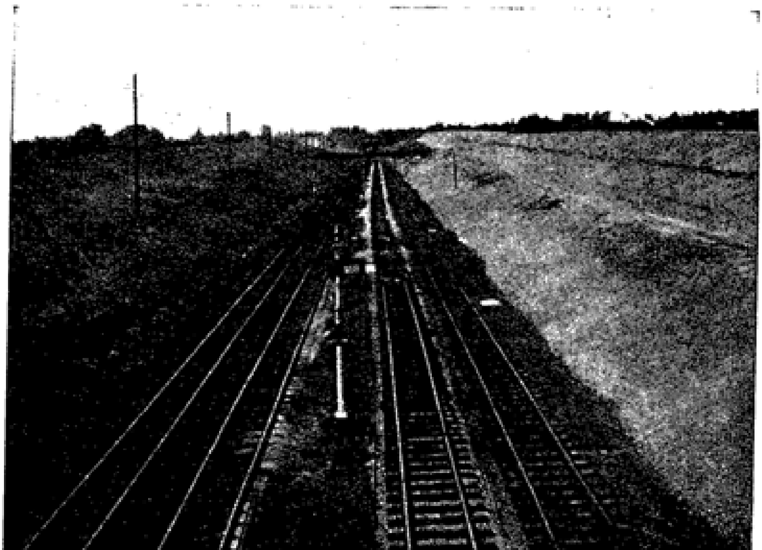


FIG. 23.—Pilning Cutting. Completion of Widening of Cutting. (Problem 40.)

ings. In such cases the following matters must be considered where clay or sandy clay soils exist :—

(*a*) The foundation for the stone-pitched face should be taken down to a sufficient depth below the circular arc of failure.

(*b*) The pitching should be designed with adequate stone-rubble-filled counterforts, which may be approximately 3 ft. wide and 5 ft. deep at intervals of 20 to 25 ft., with weep-pipes and a properly designed drainage system to avoid any possibility of water collecting in pockets under the stone pitching.

(*c*) The stone pitching should not in any case be carried up to the full height of the slope, but should be curtailed at a

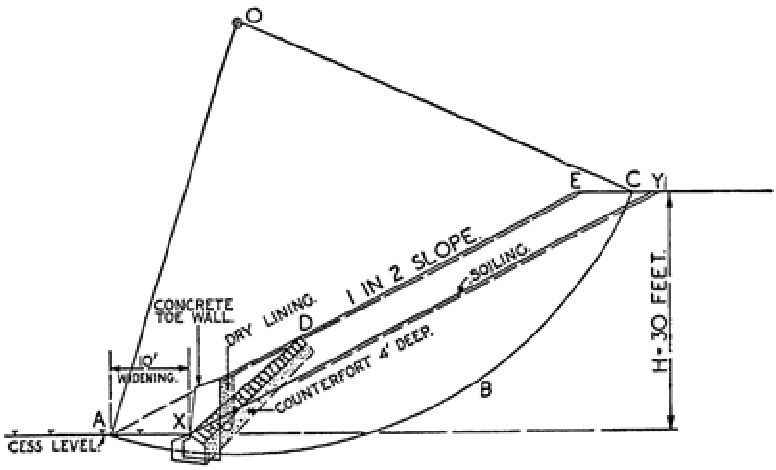


FIG. 24.—Toe Wall and Stone-pitched Slope. (Problem 41.)

point lying vertically above the centre of gravity of the segment of soil likely to slip according to the circular arc theory.

(*d*) The pitching should be constructed with the heaviest mass at the foot, and the base designed with greater thickness than at the top.

Problem 41. A cutting 30 ft. deep has a slope of 1 in 2, giving a factor of safety of 1·5 for a clay of shear strength of 570 lb. per sq. ft. and an angle of internal friction of 5°. It is proposed to widen the cutting by 10 ft. Design and compare methods of widening by means of a toe wall, stone pitching and cutting back the existing slope.

With reference to Fig. 24, to accommodate a widening of 10 ft.

would necessitate cutting back the slope to the line XY and making provision for soiling and seeding. Excluding the cost of purchase of additional land, the cost of this scheme would be approximately £65 per lin. yd. of cutting.

The provision of a concrete toe wall to a height of 7 ft. above cess level and from 3 ft. to 5 ft. thick will now be considered. The circular arc of failure is drawn as ABC. The foundation of this wall is taken down below this line to a depth of 3 ft. 6 in. below cess level. Dry stone lining with open-jointed drain- and weep-pipes are provided as indicated in Fig. 24. The cost of this work is about £61 per lin. yd. at comparative prices.

The third proposal for the provision of stone pitching will now be investigated. The pitching must not be taken any higher than the point D (*i.e.*, half the length of the slope AE). The thickness of the pitching would be about 1 ft. 6 in. at D, increasing to 3 ft. at the foot at X. Counterforts about 4 ft. deep are provided at intervals of 21 ft. (see Table 5). The concrete foundation to the foot of the pitching must be taken down below the circular arc ABC to a depth of 3 ft. 6 in. below cess level. The comparative cost of this scheme amounts to £57 per lin. yd. of cutting.

Earth-Slides in Cuttings

In sand slopes there is a tendency for particles to slip over each other, due to the increase in cohesion with depth, and generally sand-slides are surface slips. With clays, cohesion does not increase with depth, whilst the slip force does, and clay-slides are of the circular-arc type, with the characteristics that a tension crack occurs at the top of a slope and a heave takes place at the toe.

The primary cause of earth-slips is the infiltration and absorption of water in the bank, and especially is this so with clay. When the moisture in the clay is insufficient to cause saturation of the interstices, surface tension develops and the clay particles are held together. However, when a saturated condition occurs, the molecules of clay are freed from surface tension and the structure of the clay breaks down causing the slope to become unstable. Such a condition would occur if rain fell at a higher rate than could be absorbed by the clay after a period of hot weather during which surface cracks had formed.

Slips occurring in clay cuttings are of two types :—

(1) Slips taking place during or shortly after a new cutting has been excavated are due to the low shear resistance of the clay in its natural state, and occur with soft homogeneous clays with a moisture content close to the liquid limit.

(2) Slips occurring some time after a cutting has been constructed are due to the gradual breakdown of the clay, owing to its progressive swelling and softening brought about by the percolation of water into cracks and fissures. The clay is usually not homogeneous, and solid portions of stiff clay are surrounded with a matrix of almost liquid clay.

Surface Cracks

Warning that a slip is likely to occur is usually given by the appearance of surface cracks at the top of the slope. Such fissures

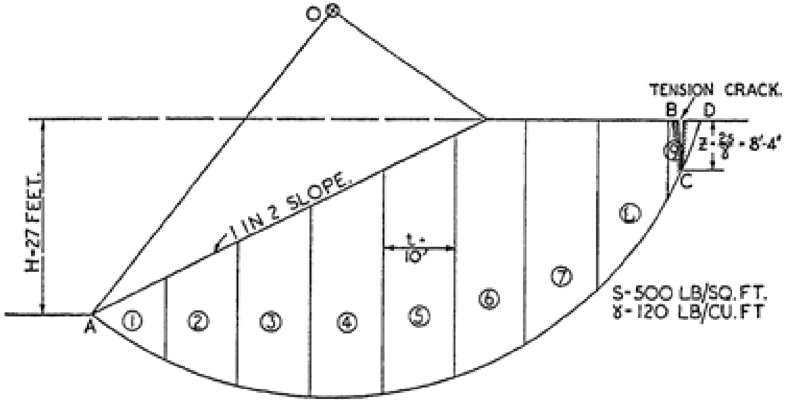


FIG. 25.—Influence of Surface Cracks. (Problem 42.)

may extend to a depth in clay soils which may be calculated as follows :—

Depth of tension crack,

$$z = \frac{2s}{\gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

In calculating the stability of a slope, especially with clays of a high liquid limit which are subject to considerable shrinkage effects, a reduction must be made in the total shear resistance along the slip circle. The following problem (42) will indicate how this may be accomplished.

Problem 42. A clay cutting 27 ft. deep with side slopes of 1 in 2 is composed of clay with a shear resistance of 500 lb. per sq. ft. and density of 120 lb. per cu. ft. Tension cracks have appeared in the upper ground surface, and a slip is likely to occur. Calculate the factor of safety, allowing for the possible depth of the surface cracks.

It will be appreciated from Fig. 25 that, due to the tension crack, the shear resistance mobilised along the slip circle will be

from A to C, which is less than the full length of the arc A to D.

The probable depth of the tension crack given by equation (29) is

$$Z = \frac{2s}{\gamma} = \frac{2 \times 500}{120} = 8' \, 4''$$

A circular arc analysis is made as follows :—

Slice No.	Tangential component, T.	Other data.
1	$-$ 3·0	ϕ = Nil.
2	$-$ 5·25	Arc length, L = 105 ft. (A to D).
3	$-$ 3·5	,, ,, L = 94 ft. (A to C).
4	+ 1·25	c = 500 lb. per sq. ft.
5	+ 7·5	t = 10 ft.
6	+13·75	γ = 120 lb. per cu. ft.
7	+16·5	
8	+14·25	
9	+ 2·7 (or + 0·8 allowing for tension crack)	

ΣT = 42·95
(or 41·05 allowing for tension crack)

The factor of safety from equation (22) making no allowance for the tension crack

$$F = \frac{Lc}{\Sigma T} = \frac{105 \times 500}{42{\cdot}95 \times 10 \times 120} = 1{\cdot}02$$

Therefore it may be assumed that the cutting slope is just stable.

Making an allowance for the tension crack necessitates reducing the length of the arc, L, from 105 ft. to 94 ft., and the factor of safety, F

$$= \frac{Lc}{\Sigma T} = \frac{94 \times 500}{41{\cdot}05 \times 10 \times 120} = 0{\cdot}95$$

This result reveals that the bank is unstable and a slip is imminent.

Remedial Measures for Slips in Cuttings

When a slip has occurred in a cutting, immediate remedial measures consist of tapping and draining away water which has collected and may be retained in the bank. Wet patches usually indicate where these accumulations of water are taking place, and at these points trenches must be cut through the face of the slip to release the water. At the same time opportunity may be taken when excavating the trenches to carry out the various quantitative tests desirable, such as shear or compression tests, liquid limit test and moisture-content determination.

The trenches may be used for the construction of stone counterforts, which have two main functions. They provide an effective method of drainage, and act as buttresses to the slope, as the friction of the slip material against the sides of the counterforts supports the bank.

The following table will give a guide for the spacing to be adopted for counterforts and lateral drains in various types of clay.

TABLE 5.

Counterfort Drains.

Type of soil.	Water content, %.	Cohesion lb./sq. ft.	Counterfort spacing.	Lateral drain spacing.
Very soft clay .	40	300	15 ft.	10 ft.
Soft clay .	35	400	17 „	12 „
Medium clay .	33 to 30	600 to 800	21 to 24 ft.	14 to 17 ft.
Stiff clay .	27 to 25	1,000 to 1,500	27 to 30 „	20 to 25 „

Counterforts. To be taken down to a depth of at least 12 ins. below the slip surface. Bottom of counterforts to be benched or stepped. Open-jointed earthenware or porous concrete pipes to be laid on bottom of trench. Width of counterfort to be from 2 ft. minimum to 5 ft. maximum. Stone or hard-core fill in trench to form counterfort with top layer of small stone.

Lateral drains. To be constructed to a herring-bone pattern as viewed on surface of slope. Depth to be from 3 ft. to 10 ft., according to length and type of soil, pipes to be provided when depth exceeds 4 ft. Width from 2 ft. to 4 ft. Hard-core fill in trench as for counterforts.

An adequate drainage system for the whole area should be designed, with due consideration given to the adjacent watershed and geological features. Percolation of water and saturation of the bank may be reduced by providing pipe or stone-filled drains along the top of the bank to act as barriers and intercept surface water draining towards the cutting. These drains should be taken down to a depth of 12 ins. below the top of the solid clay and the trench filled with hard-core or rubble, which should be surfaced with a layer of ashes and turfed.

Problem 43. A clay cutting of depth 50 ft. has side slopes of 1 in 2. The layers of clay through the cutting have varying shear strengths as indicated in Fig. 26, such information having been obtained from a number of boreholes. In particular it should be noted that a shell-bed of 2 ins. in thickness underlies the blue clay stratum at a depth of about 40 ft., and the general inclination of the beds is 6° with the horizontal.

Deformation of the surface of the slope has taken place over a

length of several chains, the upper portion has slipped downward 3 ft., fissures have appeared in the slope, a very wet patch has occurred at two-thirds the height of the slope and bulging has taken place at the toe. Investigate the stability and suggest suitable remedial measures.

With reference to Fig. 26, the failure of the slope AB in this instance is of the type described in Problem 33, and the circular

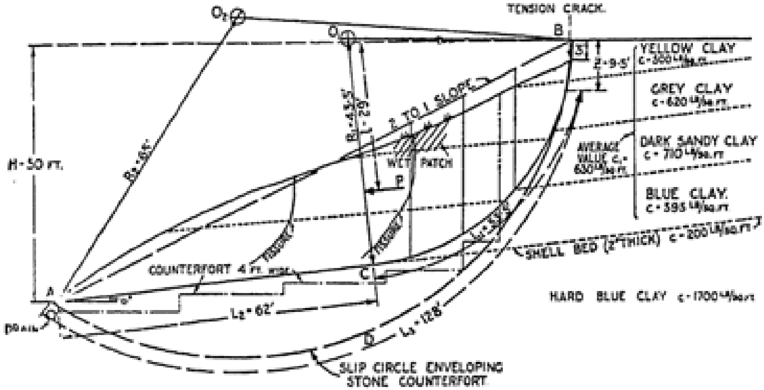


Fig. 26.—Counterfort for Clay Cutting Slip. (Problem 43.)

arc of failure tangents at the surface of the shell-bed overlying the very hard blue clay at the point C.

The depth of the tension crack,

$$\underset{\sim}{Z} = \frac{2s}{\gamma} = \frac{2 \times 500}{105} = 9' \; 6''$$

A circular arc analysis is made, and the results are tabulated as follows :—

<table>
<tr><th>Slice No.</th><th>Tangential component, T.</th><th>Other data.</th></tr>
<tr><td>1</td><td>4·0</td><td rowspan="5">L_1 = 535 ft. (allowing for $\underset{\sim}{Z}$).
c_1 = 630 lb. per sq. ft. (average value).
R_1 = 43·5 ft.
ϕ = Nil.
L_2 = 62 ft.
c_2 = 200 lb. per sq. ft.
l = 29 ft.
γ = 105 lb. per cu. ft.
t = 10 ft.</td></tr>
<tr><td>2</td><td>10·5</td></tr>
<tr><td>3</td><td>16·0</td></tr>
<tr><td>4</td><td>16·5</td></tr>
<tr><td colspan="2">ΣT = 47·0
× 10 × 105
= 49,350 lb.</td></tr>
</table>

From equation (26), the factor of safety

$$= \frac{RL_1c_1 + L_2c_2l}{R\Sigma T}$$

$$= \frac{(43{\cdot}5 \times 53{\cdot}5 \times 630) + (62 \times 200 \times 29)}{43{\cdot}5 \times 49350}$$

$$= \frac{1466640 + 359600}{2146725}$$

$$= 0{\cdot}85$$

From this result it is evident that the slope is unstable.

For the remedial measures deep counterforts should be adopted, each to be taken down to a depth of at least 12 ins. below the shell-

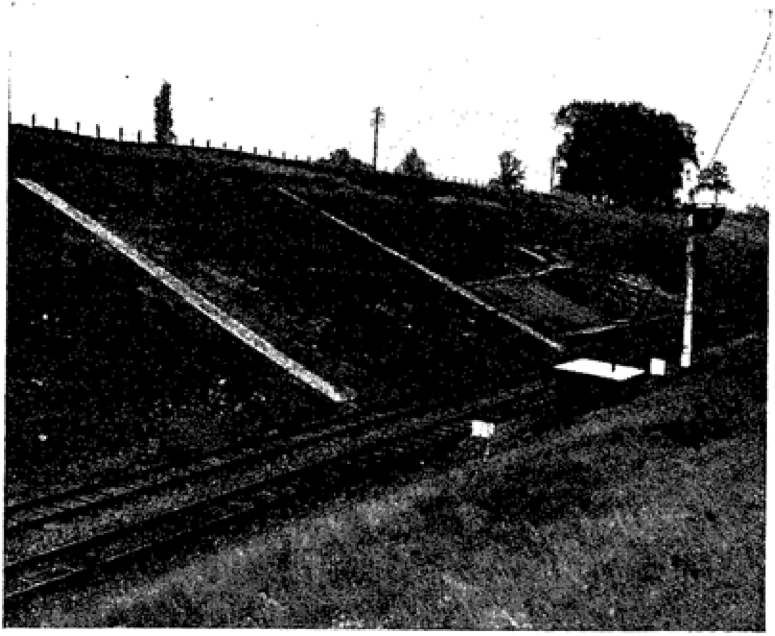


Fig. 27.—Chippenham Cutting. Construction of Deep Counterforts. (Problem 43.)

bed. The counterforts should be 4 ft. wide, and spaced at intervals of 25 ft., in accordance with Table 5. The counterforts should be stepped or benched as indicated in Fig. 26.

A circular-arc analysis adopting an arc enveloping the base of the counterfort as shown by the dashed circle ADB with centre O_2 will give a factor of safety of about 1·5.

The photograph in Fig. 27 shows the cutting referred to in this problem during the construction of the deep counterforts.

Note on Stiff Fissured Clays

London, Gault, Weald and Kimmeridge Clays are of the type known as stiff fissured clays. These clays are difficult to consider in the light of the foregoing slip-circle analysis, and must be given special consideration.

The shear resistance of the clay is usually high, but a number of random joints exist, and if a sample is allowed to fall on to a hard surface it will break into polyhedral fragments with smooth faces.

During the construction of a cutting the removal of the excavated material allows expansion and opening of these joints. As these fissures open up, water enters, and as certain surfaces of the clay in the fissures are under no load, the clay softens. A general weakening of the cutting bank occurs, and this effect is progressive.

When investigating problems involving fissured clays it is essential to ascertain the shear strength of the softened clay and use this value in the slip-circle analysis. An example occurred on the Southern Railway in 1939 in the cutting through brown and blue Weald clay on the main line south of the Sevenoaks Tunnel. The shear strength of the unfissured parts of the clay exceeded 3,000 lb. per sq. ft., and the calculated shear strength required to maintain stability was only 660 lb. per sq. ft. After the slip had taken place it was found that the shear strength of specimens of clay cut out of the slip surface varied from 1,500 lb. per sq. ft. to 280 lb. per sq. ft.

CHAPTER V

EMBANKMENTS—DESIGN AND CONSTRUCTION

Generally the methods already described in Chapters III and IV for designing and investigating the stability of cutting slopes may be applied when considering earth embankments. However, certain additional features require consideration and embankments may be divided into two types, dependent on whether or not they are required to retain water. The design of earth dams is considered to be outside the scope of the present volume but the subject may be pursued by study of the numerous American technical papers published on this subject and the standards drawn up by the Tennessee Valley Authority for the design of dams.

An embankment may fail either by slip or settlement, and it is necessary to investigate the soil properties of the ground on which an embankment is to be built as well as the soil which is to be used in its construction. The settlement of an embankment, apart from the compaction of the bank itself, may be due to the consolidation of the underlying material due to the load of the fill, and this aspect will be referred to in a later chapter dealing with consolidation. If water is to be retained, then the permeability of the soil requires careful investigation, and especially is this so with sandy type fills. In all cases, shear strength, moisture content and liquid limit tests are a necessity. In addition, the study of soil compaction is an essential in embankment design, and the principles of compaction will be briefly outlined in this chapter.

Owing to the speed at which embankments are now being constructed with modern equipment, the compaction of the fill is becoming more and more important, and enables as dense a fill to be formed at the outset as would be obtained after years of settlement and use.

Soil Properties.

A brief review of soil properties will now be given in so far as they affect embankment construction.

An embankment built with non-cohesive or granular soil will be stable if its slopes do not exceed the angle of repose of the material, and this holds good for any height of the embankment.

Cohesive soils, however, have a critical height dependent on

the shear strength and angle of slope, and should this height be exceeded the slope becomes unstable.

Clays of low liquid limit may be easily compacted, and an increase in shear strength is developed, but those of high liquid limit consolidate slowly under load, have a low shear strength and do not readily develop an increased shear resistance.

During the construction of an embankment, clays with a low liquid limit become reduced to slurry during heavy rain and traffic, and it is essential to carefully control drainage and compaction. Clays of high liquid limit are less sensitive to weather conditions,

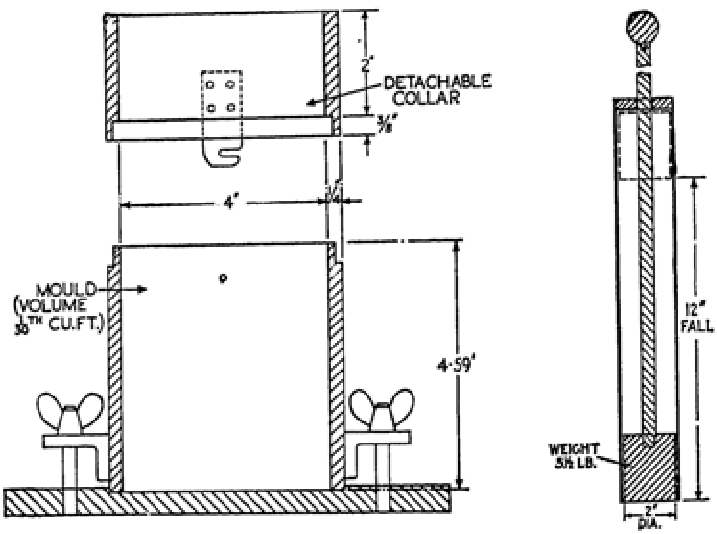


Fig. 28.—Standard Compaction Cylinder and Rammer.

but they should be avoided in the construction of high embankments, and where they exist at the site of a new embankment they are liable to deform under load.

Soil Compaction

Two classes of tests are necessary; one for determining the dry density of the soil after a standard amount of compaction has been applied, and the other for measuring the density of the soil in the field.

The state of compaction is expressed as the "relative compaction," and is the percentage ratio of the field density to the maximum density as determined by standard compaction. The percentages for relative compaction are high, since the initial

relative compaction is of the order of 80 per cent. Generally, a compaction between 90 and 97 per cent. is desirable, depending on the type of embankment to be constructed.

The standard apparatus for performing a given amount of compaction is shown in Fig. 28, and consists of a cylindrical metal mould containing 1/30th of a cubic foot and provided with a detachable collar and base. The soil is compacted in the mould by ramming with a standard $5\frac{1}{2}$-lb. rammer of 2-ins.-diameter

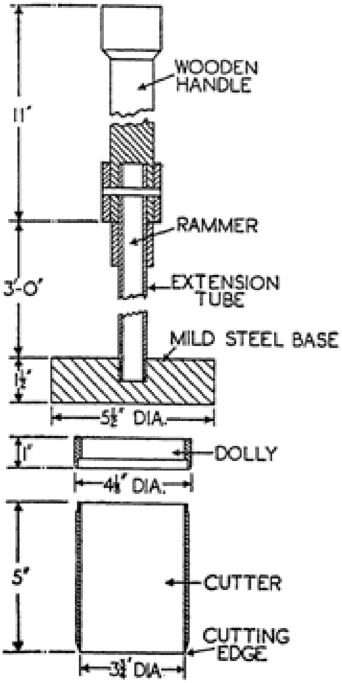


Fig. 29.—Core-cutter for Clay Soil.

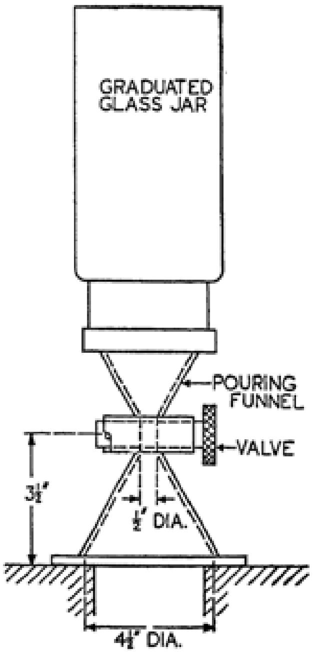


Fig. 30.—Sand Bottle for Granular Soil.

face. The rammer has a fall of 12 ins., controlled by a tubular sleeve, and the soil is compacted in three equal layers with 25 blows to each layer. The collar is removed, the sample trimmed level with the top of the cylinder and moisture content and density tests executed.

The field-density test consists of taking a sample of soil from the site, weighing and determining its moisture content. In clay soils a cutter, as shown in Fig. 29, is forced into the soil and the specimen trimmed and weighed. When dealing with granular soils, a sand-bottle is used, as indicated in Fig. 30. A specimen is

taken from the site, and the resulting hole filled with sand from the bottle, which is graduated, thus enabling the volume to be easily determined.

With a given amount of compaction there exists for each soil a moisture content, termed the " optimum moisture content," as shown in Fig. 31, from which a maximum dry density is obtained. This is due to the fact that when the moisture content is low the soil is stiff and difficult to compress; as the moisture content in-

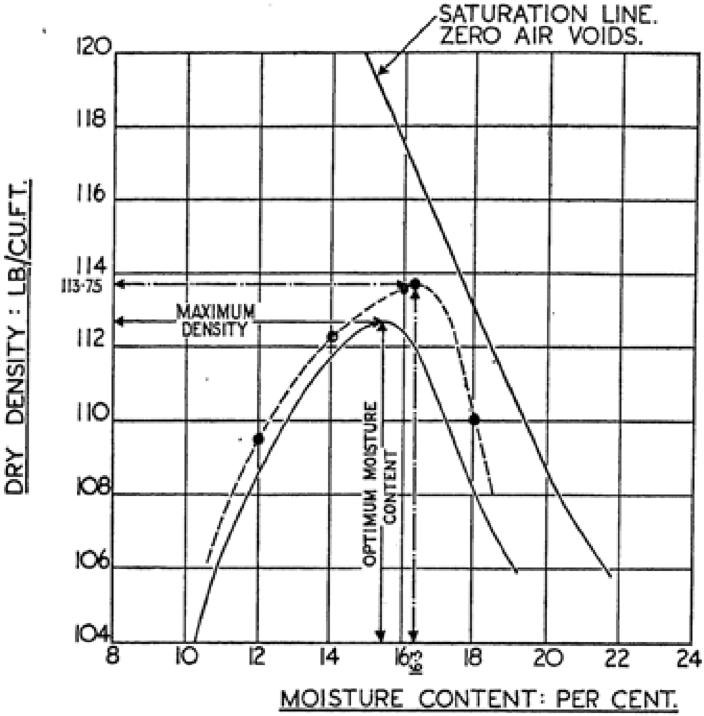


FIG. 31.—Moisture/Density Curve. (Problem 44.)

creases, so the water acts as a lubricant, the soil softens, becomes workable and a high density is obtained. Further moisture tends to keep the particles apart, prevents decrease in air content although the total voids increase, hence the dry density of the soil becomes less.

Problem 44. A sample of clay with liquid limit of 29 per cent. and plasticity index of 7 per cent. is subjected to a compaction test with the following results :—

Moisture content, per cent.	12·0	14·0	16·0	18·0
Dry density, lb./cu. ft.	109·5	112·5	113·5	110·0

Ascertain the optimum moisture content and maximum density. If the field density after compaction is 105·79 lb. per cu. ft., what is the relative compaction?

From Fig. 31 the results (which are plotted and shown by a dash-line) indicate a maximum density of 113·75 lb. per cu. ft. at an optimum moisture content of 16·3 per cent.

If the field density is 105·79 lb. per cu. ft., then the relative compaction

$$= \frac{105\cdot 79}{113\cdot 75} \times 100 = 93\%$$

Shear Strength.

An increase in shear strength occurs with compaction, and in the case of sands it is found that the strength of a thoroughly compacted sand may be double that of a loose sand. Clays have a variation in shear strength increase due to their types, cohesion and angle of internal friction. Shear-box tests with compacted samples will give the necessary data for design.

Problem 45. An embankment is to be built with clay to be excavated from a borrow pit. It is specified that the material will be compacted by rollers to result in a relative compaction of 97 per cent. (Assume the clay has an angle of internal friction of 5°.) Compaction tests give the following results :—

Moisture content, per cent.	12·0	14·0	16·0	18·0	20·0
Dry density, lb./cu. ft.	108·2	110·0	111·8	110·3	107·1

Ascertain the probable density of the fill in the completed embankment. Shear tests are performed with standard compacted samples, and the results are as follows :—

Moisture content, per cent.	12	14	16	18
Shear strength, lb./cu. ft.	630	720	648	504

Design the embankment for a height of 30 ft. with a factor of safety of 1·5 and compare with the profile for an embankment composed of uncompacted fill. The shear strength of the uncompacted fill is 560 lb. per sq. ft.

From a graph of the compaction test results as shown in Fig. 32, it is evident that the optimum moisture content is 16·5 per cent. and the maximum density is 112 lb. per cu. ft.

If the relative compaction is 97 per cent., then the probable density of the filling in the field

$$= \frac{97 \times 112}{100} \text{ lb./cu. ft.} = 108\cdot 64 \text{ lb./cu. ft.}$$

This would be equivalent to a compaction test of 15 blows instead of 25 blows, or a value comparative to a moisture content of 12·7 per cent.

The shear tests are plotted in Fig. 33, from which it is apparent that the shear strength of the fill compacted to give a relative compaction of 97 per cent., or an equivalent value for a moisture content of 12·7 per cent., is 667 lb. per sq. ft.

From Taylor's curves, given in Fig. 16, the safe slope allowing a factor of safety of 1·5, angle of internal friction of 5° and weight of soil 108·64 lb. per cu. ft., is found to be 44°.

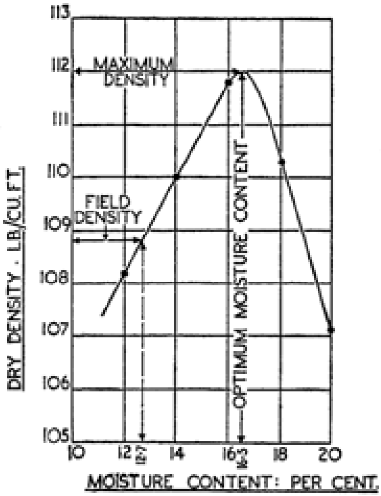


Fig. 32.—Compaction Test. (Problem 45.)

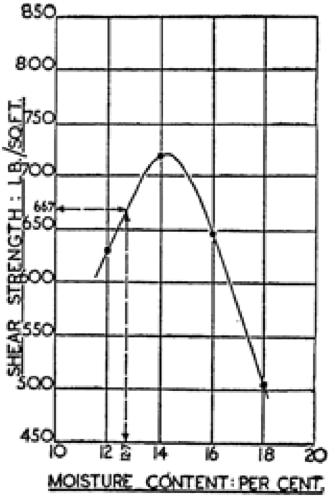


Fig. 33.—Shear Test. (Problem 45.)

The uncompacted fill has a shear strength of 560 lb./sq. ft., related to a moisture content of 10·6% or a dry density of 106·8 lb. per cu. ft. and from Fig. 16 the safe slope is 25°.

Methods of Compaction in the Field

Compaction may be carried out in the following ways :—

(1) Pressure by means of smooth steel rollers, sheepsfoot rollers, tractors and construction equipment. This type is effective with cohesive soils, but not on loose, non-cohesive soils.

(2) Impact by means of pneumatic or internal-combustion rammers and dropping-weight type of equipment. This type is effective on both cohesive and non-cohesive soils.

(3) Vibration which is very effective on non-cohesive soils, and particularly on loose sands, but it is ineffective on cohesive soils.

Sheepsfoot rollers produce highest densities, and smooth rollers are effective provided the thickness of the layers of cohesive soil to be compacted are shallow and do not exceed 8 ins. In recent works a relative compaction up to 93 per cent. has been obtained by careful control of traffic of the construction equipment required for the works.

The following notes will form a guide to the methods most suitable for various types of soil :—

(1) Silty and heavy clays should be compacted in layers from 6 to 8 ins. deep by means of 8-ton sheepsfoot rollers or construction traffic.

(2) Firm sands may be compacted in 8 to 12-ins. layers with a 5-ton smooth roller or construction equipment.

(3) Gravel, shingle or loose sand, the latter preferably damp, should be deposited in 12-ins. layers and compacted with tractor equipment.

(4) Fragmental rock (fragments not to exceed 4 ins.) may be laid in layers from 12 to 24 ins. or more, and consolidated by heavy smooth rollers or heavy construction equipment.

Table 6 briefly sets out the relation between compaction and the characteristics of soils.

TABLE 6.

Characteristics and Compaction of Soils.

Quality of soil as fill.	Compacted soil (Maximum dry density).	Liquid limit.	Plastic limit.
	Lb. per cu. ft.		
Very poor . .	70 to 100	Greater than 65%	Greater than 22%
Poor . . .	100 to 110	50 to 65%	19 to 22%
Fair . . .	110 to 120	32 to 50%	16 to 19%
Good . . .	120 to 130	24 to 32%	14 to 16%
Very good . .	Over 130	Less than 24%	Less than 14%

For the construction of embankments over 50 ft. high the dry density of the compacted material should not be less than 120 lb. per cu. ft. Gravel or sand-type soils are preferable for such embankments, but if clay is used it must be compacted to a high

relative compaction not less than 95 per cent. In any embankment construction the dry density must not be less than 90 lb. per cu. ft.

During the compaction of the soil, if it is too dry, it can either be wetted at the borrow pit or sprayed with water and mixed in place with ploughs and harrows. When the soil is too wet it may be dried by harrowing and exposing to the air under favourable weather conditions, but during adverse conditions on clay sites it may be necessary to prohibit construction, as the soil is too soft to be compacted. At 1947 costs compaction varied between $\frac{1}{2}d.$ and $2\frac{1}{2}d.$ per cu. yard of soil treated, dependent on the fill material.

Compaction may be measured during construction by means of a Proctor needle, as indicated in Fig. 34. The needle is forced

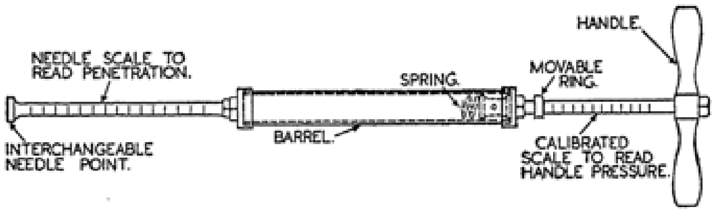


FIG. 34.—Proctor Needle.

into the soil at the rate of $\frac{1}{2}$ in. per second, and readings in lb./sq. in. are made according to the degree of softness of the soil. The tip is interchangeable, and different sizes from 0·01 to 1 sq. in. are used, according to the soil being tested. The plasticity reading is made on a sample of the soil in a compaction cylinder and the result compared with field tests; they should furnish the same readings.

During construction the material will probably have a percentage of air voids of about 5 per cent., and rainfall may cause water to be absorbed with no volume change. Under these conditions the value of dry density may be obtained by immersing the compaction cylinder in water after standard compaction has been made, and the Proctor needle may be used in this instance for comparison with field compaction.

Compaction of Granular Materials.

The most important factor controlling the density of granular materials is the sieve analysis. The moisture/density curve for such materials gives a minimum density at a moisture content between the dry, loose state and the optimum moisture content. The increased density as the material becomes very dry and loose

is due to the reduced cohesion of the particles, and this effect is usually known as " bulking."

When shear tests are performed on dense sands, an expansion takes place in the shear-box, causing a temporary increase in shear strength, whereas fine sands become compacted during a shear test, with a consequent decrease in shear strength.

Compaction of Chalk.

Compaction tests have been undertaken with chalk filling, and although partial crushing of the material occurs, the general form of the moisture/density curves follow those of other soils. When larger lumps of chalk are present the maximum dry density decreases and the optimum moisture content increases slightly.

An economy in embankment design may be achieved by providing a toe of chalk to the bank, as indicated in the following Problems Nos. 46 and 47. The toe may be of other material than chalk with a composition of stones, gravel or sand, and such construction enables either a softer material to be tipped to form the bank or a steeper side slope to be adopted.

Problem 46. A railway embankment is to be constructed 32 ft. high with a clay fill weighing 120 lb. per cu. ft., and having a shear strength of 460 lb. per sq. ft., with an angle of internal friction of 5°. What angle of slope is necessary to ensure a factor of safety of 1·5?

If a toe to the embankment is constructed consisting of consolidated chalk with a height of 8 ft., top width of 4 ft. and side slopes $1\frac{1}{2}$ to 1, what angle of slope would now be required for the factor of safety of 1·5? Compare the two schemes.

With reference to Fig. 18, the safe slope for an embankment tipped to a height of 32 ft. with this clay is 3 to 1. Therefore, the total width of ground required from the top of the slope is 96 ft., as indicated in Fig. 35 from A to B.

If a chalk toe is constructed, the height of the embankment is reduced in effect by the height of the toe, and becomes 24 ft. This is due to the fact that the circular arc of failure does not pass through the chalk of high shear strength, but is situated above it, as shown by DEF in Fig. 35.

It should be noted that assuming the original ground consists of material of shear strength greater than the new fill, then the slip circle will not pass under the toe; but if this is not the case, then an analysis must be made for the possibility of the critical slip circle being situated under the chalk toe.

From Fig. 18, the safe slope for an embankment 24 ft. high consisting of the above clay fill is 2 to 1. The total width in this

instance from the top of the slope is 64 ft., indicated from G to B. This results in the saving of the width of land to be purchased amounting to 32 ft.

A circular arc analysis is taken out to check this result, and the tabulated data follows :—

Slice No.	Normal component, N.	Tangential component, T.	Other data.
1	5·0	− 2·6	L = 86 ft.
2	13·5	− 3·2	
3	20·5	− 0·6	c = 460 lb. per sq. ft.
4	24·0	4·7	
5	24·2	10·7	ϕ = 5 degrees.
6	17·0	13·5	
7	5·7	5·5	w = 120 lb. per cu. ft.
	112·9 × 120 × 10 = 134,480 lb.	28·0 × 120 × 10 = 33,600 lb.	t = 10 ft.

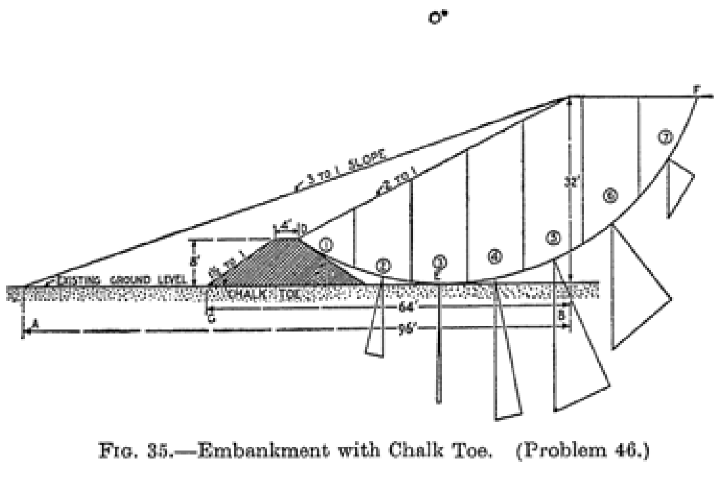


FIG. 35.—Embankment with Chalk Toe. (Problem 46.)

From equation (22) the factor of safety

$$= \frac{Lc + \tan\phi \Sigma N}{\Sigma T}$$

$$= \frac{(86 \times 460) + (0{\cdot}0875 \times 134480)}{33600}$$

$$= 1{\cdot}51$$

Comparing the two schemes, the first proposal requires about 53 cu. yds. of fill greater than the second proposal, and allowing for additional soiling and seeding, the cost will be an additional £9 10*s*. per lin. yard of embankment. The filling required for the toe will be more expensive, resulting in an additional cost of £1 10*s*. per lin. yard. Therefore, the net saving in providing the chalk toe will be £8 per lin. yard, together with a smaller area of land to be purchased.

Problem 47. An existing embankment 27 ft. high, with side slopes of 1 in 2 is to be widened 28 ft. on one side. The original ground and the existing embankment consist of hard gravel and hard clay respectively, both having shear resistances exceeding

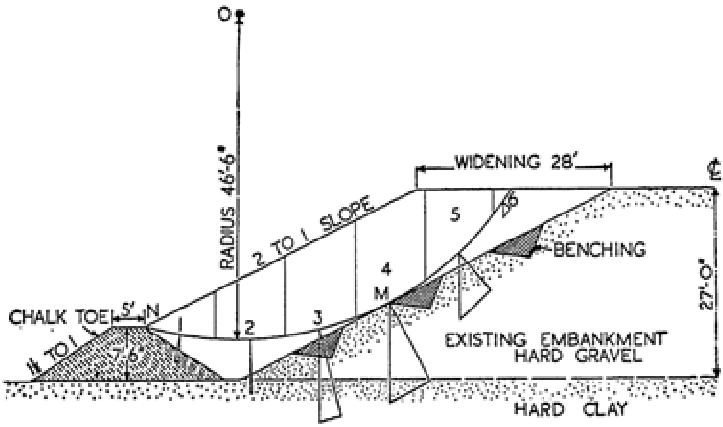


FIG. 36.—Embankment Widening with Chalk Toe. (Problem 47.)

1,000 lb. per sq. ft. It is proposed to provide a chalk toe to support the new clay fill to be used in the widening, which is to be tipped to a 1 in 2 slope. The chalk toe is to be 7·5 ft. high, top width of 5 ft. and 1½ to 1 side slopes.

Calculate the shear resistance of the clay required for this work, allowing a factor of safety of 1·5.

The conditions relating to this problem are indicated in Fig. 36. It will be evident that the slip circle will not pass through the existing embankment slope of high shear resistance, but will tangent this surface at the point M. Furthermore, the arc will not pass through either the original ground or the chalk toe, but will pass above the latter to the point N.

A circular arc analysis is made as follows :—

Slice No.	Normal component, N.	Tangential component, T.	Other data.
1	3·8	− 0·8	L = 61 ft.
2	7·5	0·3	F = 1·5 (factor of safety).
3	12·5	3·3	ϕ = zero.
4	12·5	6·7	w = 120 lb. per cu. ft.
5	7·0	6·5	t = 10 ft.
6	0·3	0·4	
	43·6 × 120 × 10 = 52,320 lb.	16·4 × 120 × 10 = 19,680 lb.	

Factor of safety (from equation 22)

$$F = \frac{Lc + \tan\phi\Sigma N}{\Sigma T}$$

$$1{\cdot}5 = \frac{(61 \times c)}{19680}$$

$$c = 480 \text{ lb. per sq. ft.}$$

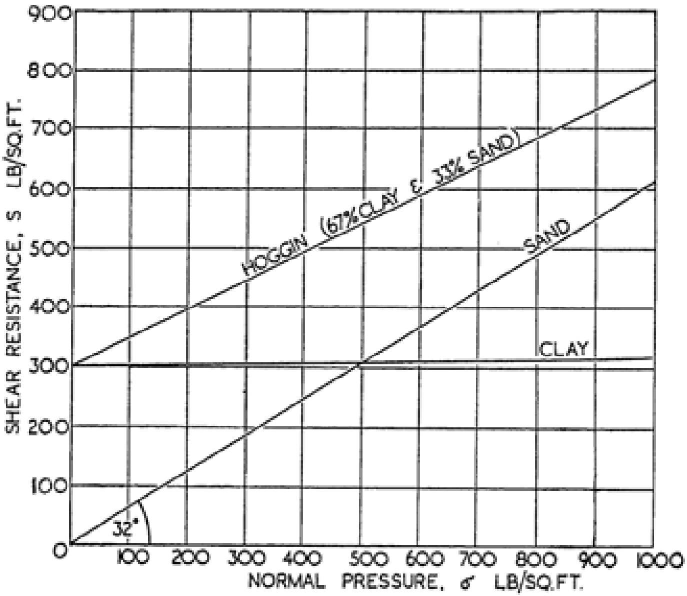


Fig. 37.—Shear Strength of Hoggin, Sand and Clay.

Therefore the clay to be used for the widening of the embankment must have a shear resistance of not less than 480 lb. per sq. ft. in order to maintain a factor of safety of 1·5.

(It should be noted that when tipping is carried out alongside an old embankment it is necessary to cut benchings in the existing slope, as shown in Fig. 36, in order to break up the plane of weakness between the new and existing work.)

Admixture of Soils.

A mixture of sand and gravel with clay will result in a material which will exhibit both cohesion and friction to a marked degree without consolidation. Such a soil mixture is obviously ideal for the construction of embankments, especially for those retaining water, such as dams.

The graph in Fig. 37 shows the relations between shear strength and normal pressure for sand, clay and a mixture of 33 per cent. sand with 67 per cent. clay. (This mixture of soils is known as hoggin.) The increase in shear strength is small below a 30 per cent. addition of sand or gravel, but over this percentage the strength of the soil mix rises to a maximum when the clay particles just fill the voids in the sand and gravel. A soil mix of this description has been used in the construction of a storage reservoir at Chingford, Essex.

Problem 48. It is proposed to construct a flood-bank 20 ft. high with a mixture of gravel, sand and clay. The clay has a cohesion of 300 lb. per sq. ft. and an angle of internal friction of 5°. The sand-and-gravel mixture has an angle of internal friction of 33°. The sand, gravel and clay are mixed in the proportions of 35 per cent. sand and gravel to 65 per cent. clay, and the density is 110 lb. per cu. ft. Tests with this mixture reveal that an angle of internal friction of 25° exists.

Compare the profiles of the embankment for fills of sand and gravel, clay and hoggin, adopting a factor of safety of 1·5. (Use Taylor's curves given in Fig. 16.)

With a fill composed of sand and gravel the angle of the side slope must be less than the angle of internal friction of 33°, and a slope of $1\frac{3}{4}$ to 1 or 30° should be adopted.

Considering a clay fill, from equation (27) the "stability number,"

$$N = \frac{c}{whF} = \frac{300}{110 \times 20 \times 1{\cdot}5} = 0{\cdot}10$$

From Fig. 16 the safe angle of slope for $N = 0{\cdot}10$ and $\phi = 5°$ is 25°, or slightly flatter than a 1 in 2 slope.

If the fill consists of hoggin with a cohesion of 300 lb./sq. ft. and angle of internal friction of 25°, then with reference to Fig. 16, for the stability number of 0·10, the safe angle of slope is given as 67°.

The comparison of these three slopes clearly indicates the scope of embankment design with varying types of fill. It can be appreciated that it may be more economical to obtain an expensive filling with a high shear value in order that slopes may be tipped safely to a steep angle, thereby saving on the quantity of filling required and the area of land to be purchased.

CHAPTER VI

RETAINING WALLS

The design of retaining walls may be conveniently divided into two stages: (i) the calculation of earth pressures, and (ii) the dimensioning of the wall to resist these pressures. In this volume, the authors will deal chiefly with the calculation of earth pressures, as the second stage involving the design of the wall itself follows standard methods which have been adequately dealt with in engineering text-books and papers.

However, a certain amount of confusion still exists among engineers regarding the formulæ applicable for earth-pressure calculations, and this is due, no doubt, to the many theories which have been published on this subject. It is hoped that this brief *résumé* of retaining wall problems contained in Chapters VI and VII will help to clarify the position which exists at the moment with our present state of knowledge.

Earth Pressures

Earth pressures on a retaining wall are of two kinds, known as "active earth pressure" and "passive earth resistance."

Active earth pressure is the pressure exerted by the earth mass on the back of the wall at the instant immediately before a shear plane is developed within the back-fill, and a sliding wedge starts moving downwards and outwards.

Passive earth resistance is a force opposed to a movement of the wall against the earth masses at the instant immediately before a sliding wedge commences moving upwards between the back of the wall and a surface of rupture. The passive resistance can never be larger than the force initiating the movement of the wall against the earth mass.

There are several theories for assessing both active earth pressure and passive earth resistance, and, generally, the surface of failure is assumed to be a plane. Recent experiments have indicated that for cohesive soils this is not correct, and failure occurs on a curved surface, but for practical considerations this is unimportant.

As mentioned in previous chapters, cohesionless (or granular) soils and cohesive soils require different treatment. In all calculations it is necessary to obtain certain data, including the following :—

The unit weight of the soil	$= \gamma$ lb. per cu. ft.
The angle of repose	$= \alpha$
The angle of internal friction	$= \phi$
The apparent cohesion	$= c$ lb. per sq. ft.
The friction angle between wall and soil	$= \delta$

The angle of repose for a granular soil is equal to the angle of internal friction, but with a cohesive soil this angle varies with the height of the soil mass and moisture content or shear strength. The values of the shear strength and angle of internal friction can be ascertained by shear tests.

Tables are available giving the earth pressure due to granular (non-cohesive) soils as a product of the unit weight of the soil and a coefficient. These tables are contained in the *Report on Retaining Walls*, published in 1934 by the Institution of Structural Engineers.

More comprehensive tables were published in the same year by the Department of Scientific and Industrial Research as *Earth Pressure Tables*, Special Report No. 24, and these tables are based on the Revised Wedge Theory. Subsequently the Institution of Civil Engineers have issued a Code of Practice for " Earth Retaining Structures ".

With regard to cohesive soils, the magnitude of the earth pressure may be calculated by the method developed by A. L. Bell (see *Minutes Proc. Inst. C.E.*, Vol. 199 (1915), Part 1), but the knowledge of the behaviour of cohesive soils is still not yet complete.

The distribution of earth pressure is assumed to be triangular but this is not always correct, particularly in cases where arching occurs or passive earth resistances are under consideration. The direction of earth pressure varies from the horizontal and is dependent on the friction between the earth and the back of the wall. The angle may vary from zero to a value approaching the internal friction of the earth mass, and usually values vary between a half and two-thirds of this angle.

The influence of a concentrated load on the back-fill can be calculated from Boussinesq's formula :—
The horizontal pressure,

$$\sigma = \frac{3 \times P \times d^2 \times h}{2\pi \times R^5} \quad . \quad . \quad . \quad . \quad (30)$$

where P = Vertical load,
d = Horizontal distance of P from back of the wall,
h = Depth at which the horizontal pressure is under consideration,
and $R = \sqrt{d^2 + h^2}$

The effect of the concentrated load diminishes rapidly with the depth.

A uniformly distributed load on the fill behind the wall may be regarded as a uniformly distributed surcharge, and may be replaced by an equivalent height of soil,

$$= h_0 = \frac{p}{\gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where p = Uniformly distributed load,
and γ = Unit weight of the soil.

The effect of the uniformly distributed load is the same over the total height.

Wall Design.

Walls retaining earth-masses may be divided into two categories: mass or gravity walls and L-shaped walls.

Mass walls resist the overturning effect of the earth-mass by their weight producing a positive moment around the toe. L-shaped walls rely on the weight of the earth-fill above the horizontal member of the wall to produce the positive moment.

In mass-retaining walls the method of design is usually that of trial and error, the cross-section being enlarged until the resultant of earth pressure and weight produces a maximum compression stress on the face of the wall not exceeding the permissible stress to which the material of the wall can be safely stressed. The factor of safety against overturning should not be smaller than 1·5, and the pressure on the soil at the toe should not exceed the permissible compression strength of the underlying soil.

L-shaped walls are constructed in reinforced concrete, and the calculation of the necessary cross-sections of concrete and steel follows the usual methods of reinforced concrete design.

Retaining Wall Problems—Cohesionless Soils

Problem 49. A retaining wall is to be constructed to retain an earth-mass 20 ft. high of granular soil with an angle of repose of 35° and a unit weight of 100 lb. per cu. ft. The top of the earth back-fill is horizontal, as indicated in Fig. 38. Ascertain the magnitude, point of application and direction of the active earth pressure.

(*a*) Coulomb's theory. From the tables given on page 23 of the

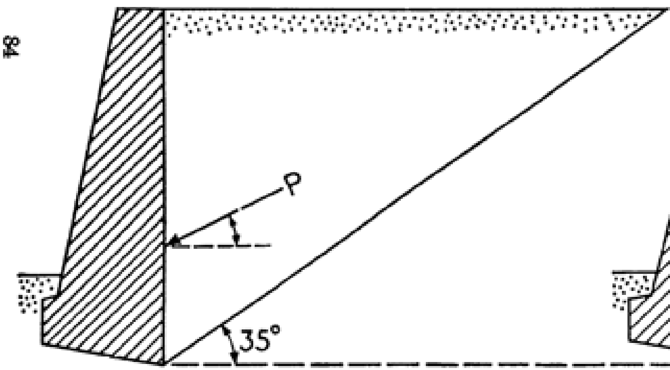


FIG. 38.—Retaining Wall. (Problem 49.)

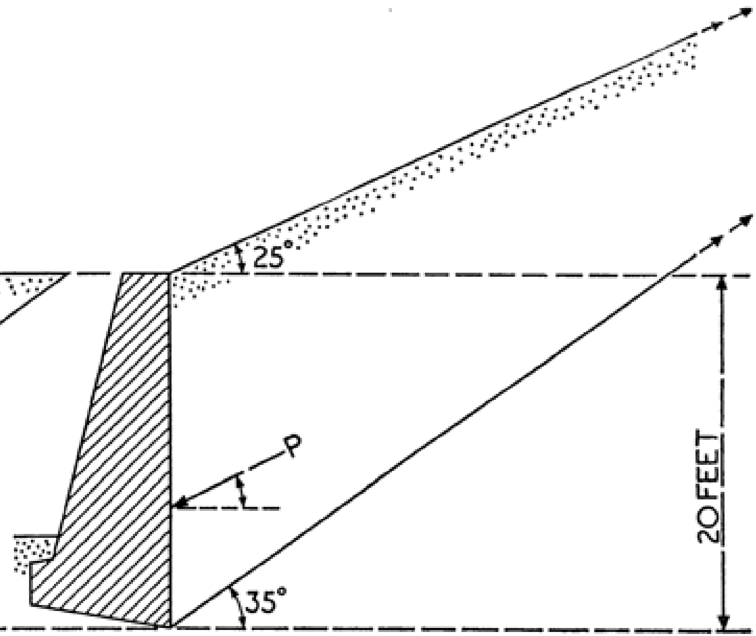


FIG. 39.—Surcharged Retaining Wall. (Problem 50.)

Report on Retaining Walls of the Institution of Structural Engineers, and using the given data, the following value is obtained :—

$$\frac{K_3H^2}{2} = 50$$

The earth pressure, $P = 50 \times 100 = 5{,}000$ lb. per lin. ft. of wall, the point of application will be at $(20' \times \frac{1}{3} =)$ 6 ft. 8 ins. above the foot of the wall, and its direction will be $(\frac{2}{3} \times 35° =)$ 23·3° with the horizontal.

(*b*) Revised wedge theory. From Table I, page 8, of the *Building Research Special Report No.* 24, for $\phi = 35°$ and $i =$ zero, $\beta = \gamma$, therefore there is only one plane of rupture at 27° 30′ from the vertical back of the wall.

Reference to Table III, page 46, for $\phi = 35°$ and $i =$ zero, the following data are obtained :—

$$\theta = 35°, A = 205 \text{ and } B = 143$$

Therefore the horizontal component,

$$H = \frac{205}{1000} \times \frac{100 \times 20 \times 20}{2} = 4{,}100 \text{ lb.}$$

and the vertical component,

$$V = \frac{143}{1000} \times \frac{100 \times 20 \times 20}{2} = 2{,}860 \text{ lb.}$$

Hence the total earth pressure,

$$P = \sqrt{4100^2 + 2860^2} = 5{,}000 \text{ lb./lin. ft. of wall.}$$

The direction of the force is $(90° - 35°) = 55°$ with the horizontal and the point of application is $(0{\cdot}4 \times 20') = 8$ ft. above foot of wall.

It will be observed from the above results that the forces obtained by these two methods are in close agreement, as the higher point of application of the earth pressure obtained by the revised wedge theory is compensated by a greater angle of the direction of this force.

Problem 50. Assume similar conditions as in the preceding Problem 49, but consider a surcharged back-fill with an angle of 25° to the horizontal above the top of the wall, as indicated in Fig. 39. Ascertain the magnitude, point of application and direction of the earth pressure.

(*a*) Coulomb's theory. From the graphs given in the *Institution of Structural Engineers' Report*, page 21, Plate 1, it is ascertained that $k_1 = 0{\cdot}37$.

The earth pressure,

$$P = 0{\cdot}37 \times \frac{100 \times 20 \times 20}{2} = 7{,}400 \text{ lb./lin. ft.}$$

the point of application will be ($\frac{1}{3} \times 20' =$) 6 ft. 8 ins. above the foot of the wall, and the direction of the force will be ($\frac{2}{3} \times 35° =$) 23·3° with the horizontal.

(*b*) Revised wedge theory. From the *Building Research Station Report*, Table I, page 8, when $\theta = 35°$ and $i = 25°$, it is ascertained that $\beta = 16° \ 16' \ 10''$ and $\gamma = 38° \ 43' \ 50''$, but as the back of the wall is vertical, $\alpha =$ zero and is less than β thus there is only one plane of rupture.

From Table III, page 44, $A = 311$ and $B = 218$; therefore, $H = 6{,}250$ lb. and $V = 4{,}360$ lb. Hence total earth pressure, $P = 7{,}625$ lb. per lin. ft. of wall, the point of application is ($0{\cdot}4 \times 20' =$) 8 ft. above the foot of the wall, and the direction of the force is ($90° - 35^6 =$) 55° with the horizontal.

Problem 51. Assume similar conditions as in the preceding Problem 50, but consider the back of the wall to be battered to an angle of 20° with the vertical towards the front of the wall, as indicated in Fig. 40. Ascertain the magnitude, point of application and direction of the earth pressure.

Revised wedge theory. From *Building Research Special Report*, Table I, page 8, for $\phi = 35°$ and $i = 25°$; $\beta = 16° \ 16' \ 10''$ and $\gamma = 38° \ 43' \ 50''$. The back of the wall is battered to $\alpha = 20°$, which is greater than β and therefore there will be two planes of rupture.

From Table III, page 44, $0 = 34° \ 41' \ 20''$, $A = 450$ and $B = 636$, hence the total earth pressure, $P = 15{,}600$ lb./lin. ft. of wall. The direction of this force will be at an angle of 34° 41′ 20″, with a vertical to the back of the wall or 54° 41′ 20″ with the horizontal. The point of application is at ($0{\cdot}4 \times 20' =$) 8 ft. above the foot of the wall.

Problem 52. Assume similar conditions as in Problem 49, but consider that a uniformly distributed load of 250 lb. per sq. ft. exists on the horizontal surface of the back-fill, as indicated in Fig. 41. Ascertain the magnitude, point of application and direction of the earth pressure.

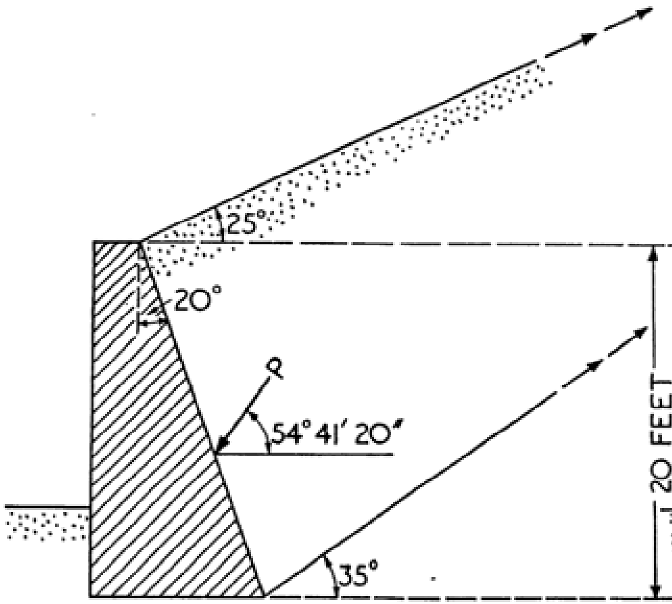


FIG. 40.—Retaining Wall with Battered Back.
(Problem 51.)

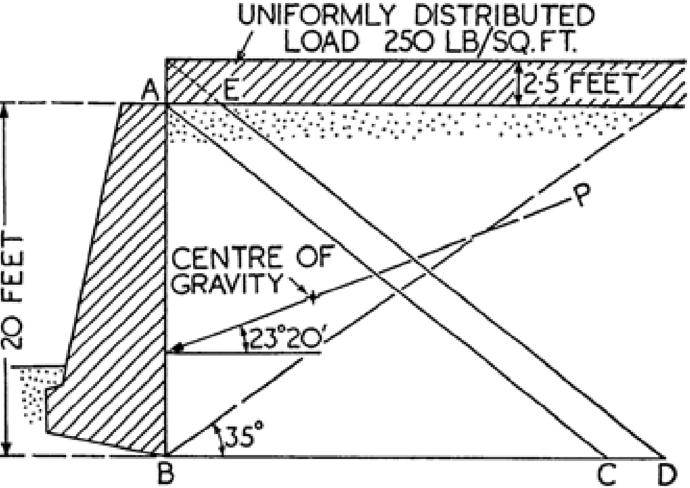


FIG. 41.—Retaining Wall, Uniformly Distributed Load.
(Problem 52.)

The equivalent height of earth by which the load of 250 lb. per sq. ft. can be replaced

$$= h_0 = \frac{p}{\gamma} \text{ (equation 31)} = \frac{250}{100} = 2{\cdot}5 \text{ ft.}$$

Therefore the total height for which the earth pressure is to be calculated is 22·5 ft.

From Plate IIIB, page 23, *Institution of Structural Engineers' Report*, the following value is obtained :—

$$= \frac{K_3H^2}{2} = 65$$

and the total earth pressure, $P = 65 \times 100 = 6{,}500$ lb./lin. ft. of wall.

The point of application of this force can be ascertained, as shown in Fig. 41, by drawing a line at an angle less than $(\frac{2}{3} \times 35° =)$ 23·3° with the horizontal and through the centre of gravity of the trapezium ABDE. This figure has sides

$$AE = 100 \times 2{\cdot}5 \times 2{\cdot}5 \text{ and}$$

$$BC + CD = (100 \times 20 \times 0{\cdot}25) + (100 \times 2{\cdot}5 \times 2{\cdot}5).$$

The direction of the earth pressure will be less than an angle of 23·3° with the horizontal.

Problem 53. Assume similar conditions as in the preceding Problem 52, but consider the earth-mass to be retained is composed of two strata, as indicated in Fig. 42. For a height of 10 ft. above the foot of the wall the fill consists of a granular soil with a unit weight of 100 lb. per cu. ft. and an angle of repose of 35°. Above this level the fill is a granular soil with a unit weight of 80 lb. per cu. ft. and an angle of repose of 25°. Ascertain the magnitude of the active earth pressure.

The earth pressure can be considered as being developed in two parts. The upper fill (weight 80 lb./cu. ft. and $\phi = 25°$) together with the surcharge; and the lower fill (weight 100 lb./cu. ft. and $\phi = 35°$) together with the surcharges of the upper fill and the uniformly distributed load.

Considering the upper fill, the equivalent height of earth replacing the uniformly distributed load

$$= h_0 = \frac{p}{\gamma} = \frac{250}{80} = 3{\cdot}13 \text{ ft.} \qquad \text{(As Equation 31)}$$

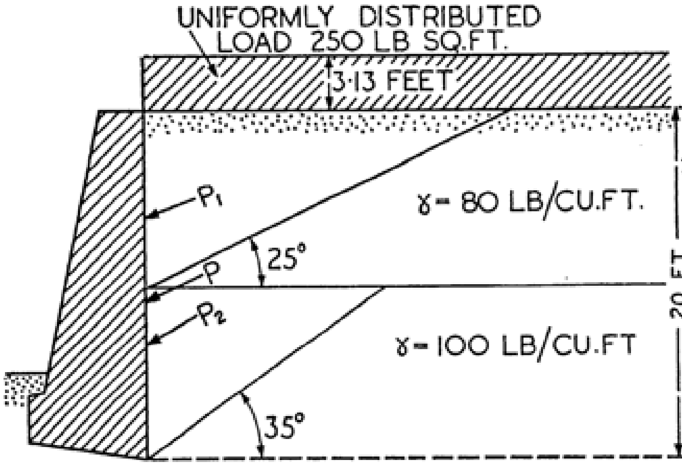


FIG. 42.—Surcharged Retaining Wall with Varying Strata. (Problem 53.)

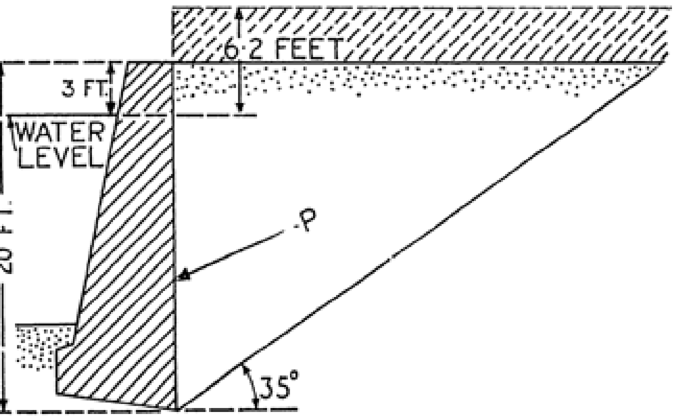


FIG. 43.—Partially Submerged Retaining Wall. (Problem 54.)

The total height for calculating the earth pressure of the upper strata is 13·13 ft. From Plate IIIB, page 23, of the *Institution of Structural Engineers' Report*, the following value is obtained :—

$$\frac{K_3H^2}{2} = 35$$

and the total earth pressure of the upper fill,

$$P_1 = 35 \times 80 = 2{,}800 \text{ lb. per lin. ft. of wall.}$$

The equivalent height of the lower fill is

$$10 + \left(10 \times \frac{80}{100}\right) + \frac{250}{100} = 20{\cdot}5 \text{ ft.}$$

From Plate IIIB, page 23,

$$\frac{K_3H^2}{2} = 55$$

Therefore, the total earth pressure of the lower fill,

$$P = 55 \times 100 = 5{,}500 \text{ lb. per lin. ft. of wall.}$$

The point of application of each of these forces can be found as indicated in Problem 52 and then combined into a single force.

Problem 54. Assume similar conditions as in Problem 49, but consider the wall to be temporarily submerged, the water rising to a level 3 ft. below the coping before sudden draw-down conditions occur, as indicated in Fig. 43. (The specific gravity of the soil grains is 2·4.) Ascertain the magnitude, point of application and direction of earth pressure.

Dry density (from equation 6)

$$= \frac{\rho}{1 + e}\gamma_w$$

therefore, $$100 = \frac{2{\cdot}4}{1 + e} \times 62{\cdot}5 \text{ or } e = 0{\cdot}5$$

Submerged density (from equation 8)

$$= \frac{\rho - 1}{1 + e}\gamma_w$$

$$= \frac{2{\cdot}4 - 1}{1 + 0{\cdot}5} \times 62{\cdot}5 = 58{\cdot}4 \text{ lb./cu. ft.}$$

Saturated density (from equation 7)

$$= \frac{\rho + e}{1 + e}\gamma_w$$

$$= \frac{2{\cdot}4 + 0{\cdot}5}{1 + 0{\cdot}5} \times 62{\cdot}5 = 121 \text{ lb./cu. ft.}$$

Before sudden draw-down takes place the earth pressure at the back of the wall can be ascertained in the manner indicated in the preceding Problem 53, the 3′ 0″ of soil above the water level can be considered as saturated and replaced by an equivalent height of

$$h \times \frac{\text{Saturated density}}{\text{Submerged density}} = 3 \times \frac{121}{58 \cdot 5} = 6 \cdot 2 \text{ ft.}$$

From the *Institution of Structural Engineers' Report*, Plate IIIB, page 23, for $H = 23 \cdot 2$ ft. and $\phi = 35°$, we obtain

$$\frac{K_3 H^2}{2} = 67 \cdot 5$$

and, therefore, the total earth pressure is

$$P = 67 \cdot 5 \times 58 \cdot 5 = \underline{3,880 \text{ lb./lin. ft. of wall.}}$$

The water pressure on both sides of the wall will be in equilibrium.

When sudden draw-down occurs the pressure at the back of the wall is increased and from the same Plate IIIB, for $H = 20$ ft. and $\phi = 35°$, we obtain

$$\frac{K_3 H^2}{2} = 50$$

and the total earth pressure is now

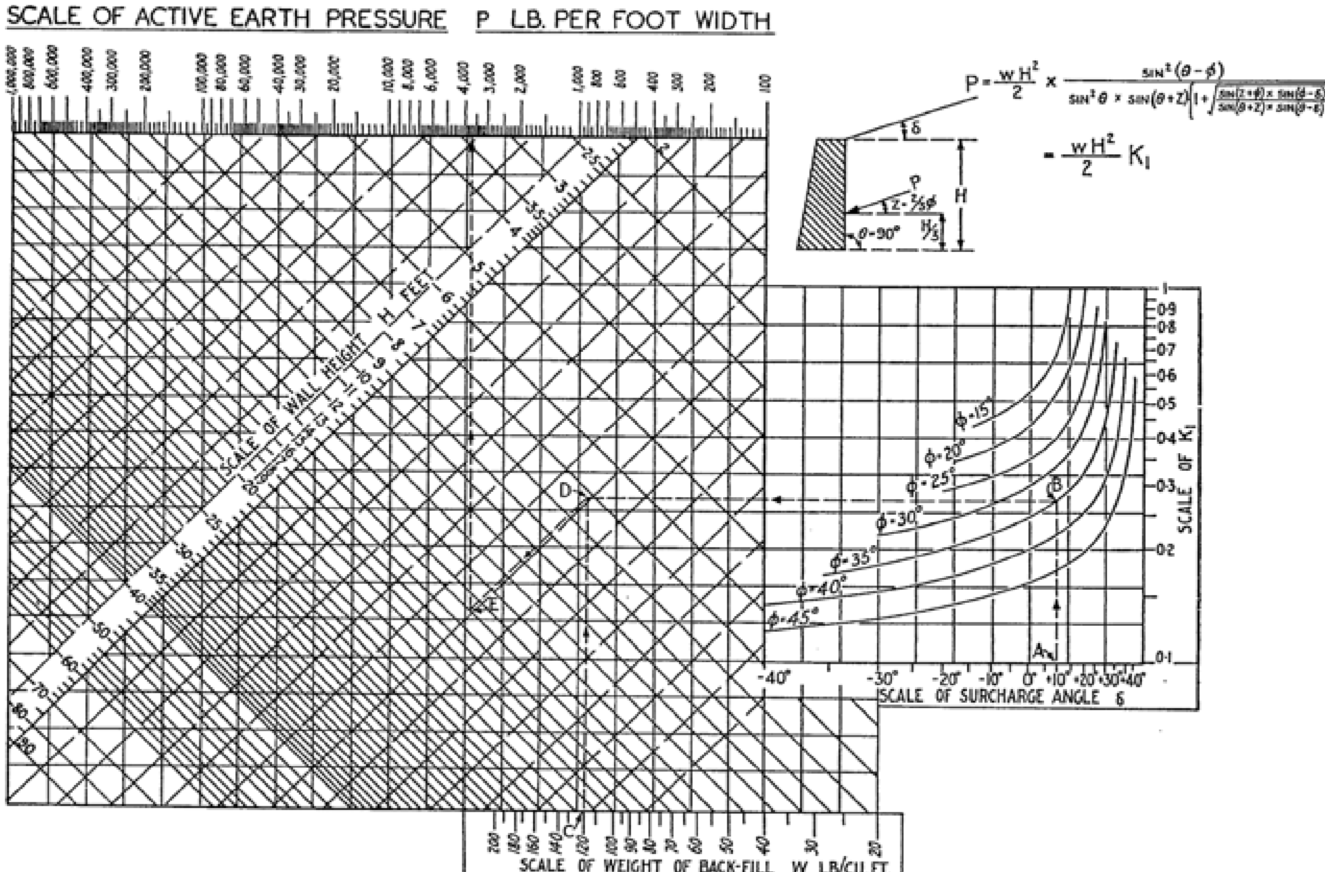
$$P = 50 \times 121 = \underline{6,050 \text{ lb./lin. ft. of wall.}}$$

This problem clearly indicates the considerable increase in earth pressure on the back of a retaining wall due to rapid draw-down conditions, especially in cases where there is inadequate provision for drainage of water from the back-fill.

Nomograms for Earth Pressures—Cohesionless Soils

The authors have devised nomograms for the rapid estimation of both active earth pressure and passive earth resistance. These are published in Figs. 44 and 45 relating to cohesionless soils, and they are based on the formulæ used in the *Report on Retaining Walls*, published by the Institution of Structural Engineers. Similar nomograms for cohesive soils will be given in Chapter VII of this volume. The following problems will illustrate the use of these nomograms.

Problem 55. A retaining wall is to be constructed to retain an earth-mass 15 ft. high of granular soil with an angle of repose of 35° and a unit weight of 120 lb. per cu. ft. The back-fill is surcharged to an angle of 10° with the horizontal. Ascertain from the nomogram in Fig. 44 the active earth pressure.



$$P = \frac{wH^2}{2} \times \frac{\sin^2(\theta - \phi)}{\sin^2\theta \times \sin(\theta + Z)\left[1 + \sqrt{\frac{\sin(Z+\phi) \times \sin(\phi - \delta)}{\sin(\theta + Z) \times \sin(\theta - \delta)}}\right]^2} = \frac{wH^2}{2} K_1$$

FIG. 44.—Nomogram for Active Earth Pressure, Cohesionless Soils.

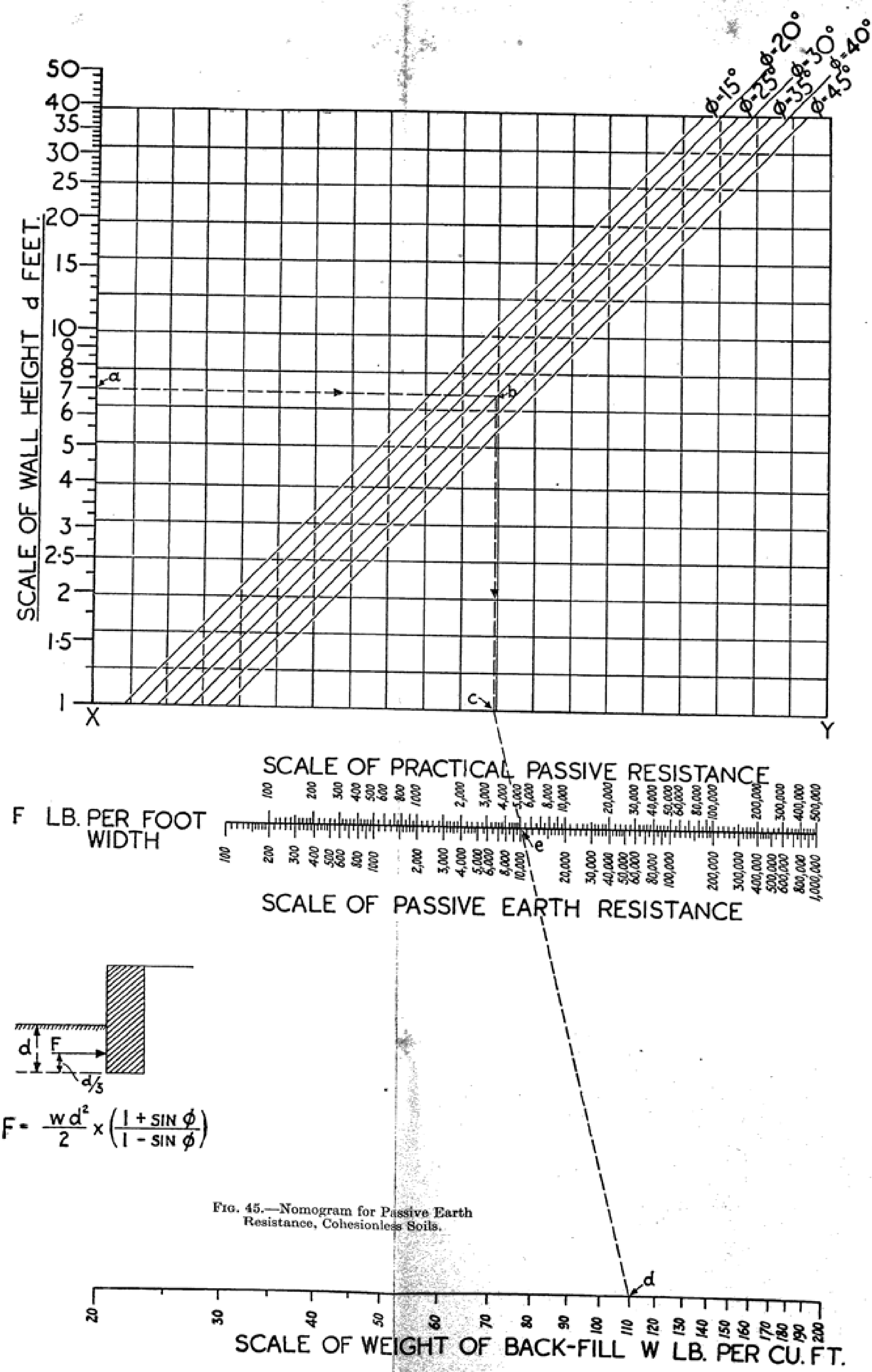


Fig. 45.—Nomogram for Passive Earth Resistance, Cohesionless Soils.

[To face p. 93.

With reference to Fig. 44, from the point + 10° on the scale of surcharge angles at A, erect a perpendicular to intersect the curve for $\phi = 35°$ (angle of repose) at the point B.

From the appropriate value on the scale for weights of back-fill, 120 lb. per cu. ft., at C, draw a perpendicular to intersect a horizontal from B, which occurs at the point D.

From D a line is projected at 45° to cut the requisite line representing the height of the wall (H = 15 ft.) at the point E. From E a vertical is drawn to the top scale reading direct the active earth pressure in lb. per lin. ft. of wall, which in this example is 3,600 lb. per lin. ft.

Problem 56. An abutment wall is founded 7 ft. below ground level in a granular soil of weight 110 lb. per cu. ft. and angle of repose of 35°. What is the passive earth resistance of the material in front of the wall?

With reference to Fig. 45, from the point d = 7 ft. on the scale for wall heights marked a, draw a horizontal until the line for $\phi = 35°$ is intersected at point b and from this point drop a perpendicular to the point c on the horizontal axis XY.

The point c is joined to the appropriate value for the weight of the back-fill, 110 lb. per cu. ft., at point d, which cuts the scale at point e, thereby giving a direct reading for the value of the passive earth resistance. This value should be halved, as indicated on the upper side of the scale, as the passive resistance is not fully mobilised until some movement of the wall occurs, and in this example the practical passive resistance is 5,000 lb. per lin. ft. of wall.

CHAPTER VII

RETAINING WALLS (*continued*)

Cohesive Soils—Active Earth Pressure

When the back-fill to a retaining wall consists of a cohesive soil, there exists a tangential force along the back of the wall partly due to adhesion and partly due to wall friction. The adhesion per unit area of wall cannot exceed the value of cohesion, c, of the clay soil. In soft clays the adhesion is likely to be equal to the full cohesion, c, but in harder clays it is considerably less. The angle of wall friction, δ, is usually small for cohesive soils, and may generally be taken as being equal to the angle of internal friction, ϕ. The shear resistance along the slip plane of the sliding wedge of back-fill is not due to friction, but to the shear strength of the clay soil, s.

Furthermore, there is a tendency for tension cracks to develop in the upper zone of the clay behind the wall, and such cracks may extend to a depth of

$$Z = \frac{2s}{\gamma} \quad \text{(See equation 29)}$$

By drawing a series of force polygons for various inclinations of the slip plane, as illustrated in Fig. 46, and plotting a graph for the active earth pressure, E, the maximum value for E can be determined. To draw the force polygon, a trial slip plane AF is taken, and the weight of the earth mass is calculated as W. This force is drawn vertically and to a scale of inches representing tons. The friction of the clay against the back of the wall, $\bar{c}$, and the shear strength along the slip plane, $\bar{s}$, are calculated and force lines drawn to represent magnitude and direction. The direction of the forces Q and E are drawn for equilibrium and scaled for magnitude. The force Q is at an angle of $(90° - \phi)$ to the slip plane, and the force E is at an angle of $(90° - \delta)$ to the back of the wall, where ϕ is the angle of internal friction and δ is the angle of wall friction. (Problems 57 and 58 will indicate the procedure to be followed.)

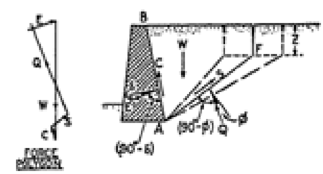


FIG. 46.—Retaining Wall. Active Earth Pressure in Clay.

This method is applicable for all conditions, such as walls with inclined back, superloading, surcharged back-fill, etc. Varying soil strata can be dealt with by regarding the upper stratum as a superload on the one below, as indicated in Problem 53, Chapter VI.

The effect of surface water or rainwater filling surface cracks should not be overlooked, as hydrostatic pressure due to a head equal to the depth of the crack may develop. The failure of a number of retaining walls with a back-fill of clay many years after construction suggests that there has been a progressive deterioration of the clay due to percolation of water in the fissures, causing swelling and finally reduction of the shear strength. The possibility of such failures could be eliminated by the construction of proper drainage with hard-core counterforts leading to weep-pipes through the wall, or the provision of an impermeable apron covering the ground surface over the whole area of the unstable wedge.

Retaining Wall Problems—Cohesive Soils

Problem 57. A retaining wall is to be constructed to retain a clay soil which has a cohesion of 400 lb. per sq. ft. and a unit weight of 120 lb. per cu. ft. The height of the wall is to be 20 ft., and the back of the wall is to be provided with a positive batter of 1 in 5. The clay back-fill is surcharged to a slope of 1 in 5 above the top of the wall, as indicated in Fig. 47. Assume the angle of internal friction is zero and that adhesion between the wall and the clay is developed to the full value of 400 lb. per sq. ft. Ascertain the magnitude, point of application and direction of the active earth pressure.

With reference to equation 29, a surface crack may develop to a depth of

$$\frac{2s}{\gamma} = \frac{24400}{120} = 6' \ 8''$$

as indicated by DF in Fig. 47.

A trial slip plane AF is drawn at an angle of

$$\alpha = 45^\circ + \frac{\phi}{2}$$

which in this case is 45°. The weight of the earth mass ABDF is calculated as W. The friction of the clay against the back of the wall is AB $\times$ c $= \bar{c}$ and the shear strength along the slip plane is AF $\times$ s $= \bar{s}$. A force polygon is then drawn for W, $\bar{c}$, $\bar{s}$, and for equilibrium there will be the reaction, Q, of the clay below AF, and the reaction of the wall, E, which will be equal to the earth

pressure reversed in direction. The direction of the force Q will be normal to AF and force E will be normal to AB. (*Note.*—If the angle of internal friction is ϕ, then the direction of Q will be at $(90° - \phi)$ to AF; and if the angle of wall friction is δ, then the direction of E will be at $(90° - \delta)$ to AB, as shown in Fig. 46.)

The completion of the force polygon enables a value for E to be ascertained. This procedure is repeated for several slip planes AF_1, AF_2, etc., and values of E are plotted to form a curve XYZ from which the maximum at Y can be determined. The angle of slip plane corresponding to Y is then the critical angle of rupture, and the value of E at this point in the present problem amounts to 13,000 lb. per lin. ft. of wall.

The direction and point of application of the earth pressure will be for practical purposes parallel to the critical angle of the slip plane AF, and through the centre of gravity of the figure ABDF, which in this instance is at an angle $\alpha = 29\frac{1}{2}°$ to the horizontal and acts at a point 6 ft. 8 ins. above the base of the wall.

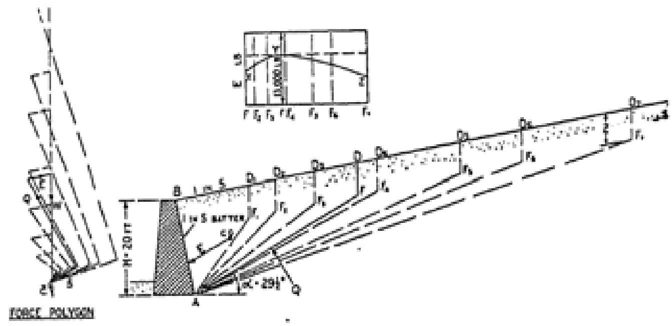


FIG. 47.—Retaining Wall. Surcharged Clay Back-fill. (Problem 57.)

Problem 58. A retaining wall is to be constructed to retain a clay soil which has a cohesion of 400 lb. per sq. ft. and a unit weight of 120 lb. per cu. ft. The height of the wall is to be 30 ft., and the back of the wall is to be provided with a positive batter of 1 in 6. The clay back-fill is level with the top of the wall. Assume the angle of internal friction is zero.

Compare the magnitude of the active earth pressure under the following conditions :—

(*a*) Adhesion between the wall and the clay is developed to the full value of 400 lb. per sq. ft. and surface cracks occur.

(*b*) Adhesion is developed as in (*a*) but no surface cracks occur.

(*c*) Adhesion between wall and clay can be neglected, but surface cracks occur.

(d) Adhesion between wall and clay can be neglected, and no surface cracks occur.

(e) Back of wall vertical, adhesion between wall and clay can be neglected, and no surface cracks occur.

(a) Assuming adhesion between the back of the wall and the clay is 400 lb. per sq. ft., then the total force along the back of the wall is $\bar{c} = 400 \times AB = 400 \times 30{\cdot}4 = 12{,}160$ lb.

The depth of the surface crack

$$Z = \frac{2s}{\gamma} = \frac{2 \times 400}{120} = 6'\ 8''$$

Taking trial slip planes AF_1, AF_2, etc., force polygons are constructed giving values of the active earth pressure, E, which are plotted as shown in Fig. 48, and the maximum value for E is 29,000 lb. per lin. ft. of wall.

The critical slip plane is AF and the centre of gravity O of ABDF is determined, thus enabling a line to be drawn through O and parallel to AF to indicate the direction and point of application of the earth pressure, E. E acts at $\alpha = 35°$ to the horizontal, and the horizontal component of E is $E_H = 23{,}800$ lb. per lin. ft. of wall, acting at a height of 10 ft. above the foundation of the wall.

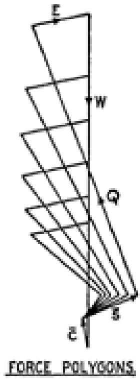
(b) Assuming surface cracks do not develop, then the shear strength along the slip plane will be increased, which will reduce the magnitude of the earth pressure E. Trial slip planes are drawn as previously indicated, and a revised series of force polygons constructed as shown in Fig. 49. The maximum value for E is 27,000 lb. per lin. ft. of wall, and the direction and point of application are found as in the previous case.

E acts at $\alpha = 29°$ to the horizontal, and the horizontal component $E_H = 23{,}600$ lb. per lin. ft. of wall acting at a height of 10 ft. above the base of the wall.

(c) Assuming no adhesion develops, but surface cracks occur, then, adopting the same method, a fresh series of force polygons are drawn as in Fig. 50, omitting the force $\bar{c}$. The maximum value for E is 37,000 lb. per lin. ft. of wall, acting at an angle $\alpha = 45°$ to the horizontal, or the horizontal component $E_H = 26{,}200$ lb. per lin. ft. of wall, acting at one-third the height of the wall above the base.

(d) Assuming both adhesion and surface cracks may be neglected, then the force polygons are constructed as in Fig. 51, which results in a maximum E amounting to 33,000 lb. per lin. ft. of wall, acting at an angle $\alpha = 37°$ to the horizontal, or the horizontal component $E_H = 26{,}400$ lb. per lin. ft. of wall, acting at one-third the height of the wall above the base.

(e) If the back of the wall is vertical and both adhesion and



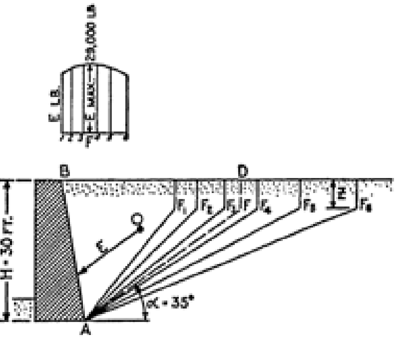
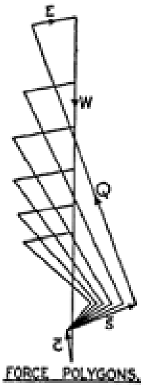


Fig. 48.—Retaining Wall. Cohesive Back-fill. (Problem 58 (*a*). Influence of Wall Adhesion and Surface Cracks.)



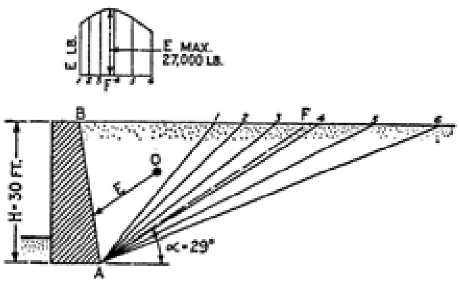
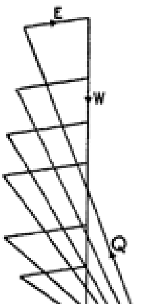


Fig. 49.—Retaining Wall. Cohesive Back-fill. (Problem 58 (*b*). Influence of Wall Adhesion Without Surface Cracks.)



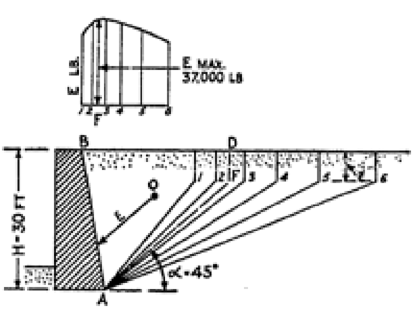


Fig. 50.—Retaining Wall. Cohesive Back-fill. (Problem 58 (*c*). Influence of Surface Cracks Without Wall Adhesion.)

surface cracks may be neglected, then the series of force polygons are drawn as indicated in Fig. 52. The maximum earth pressure E is 37,500 lb. per lin. ft. of wall, acting at angle of $\alpha = 26°$ to the horizontal or the horizontal component $E_H = 33,400$ lb. per lin. ft. of wall, acting at 10 ft. above the base of the wall.

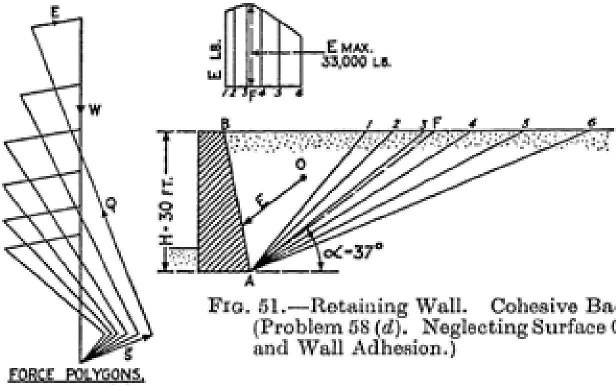


FIG. 51.—Retaining Wall. Cohesive Back-fill. (Problem 58 (*d*). Neglecting Surface Cracks and Wall Adhesion.)

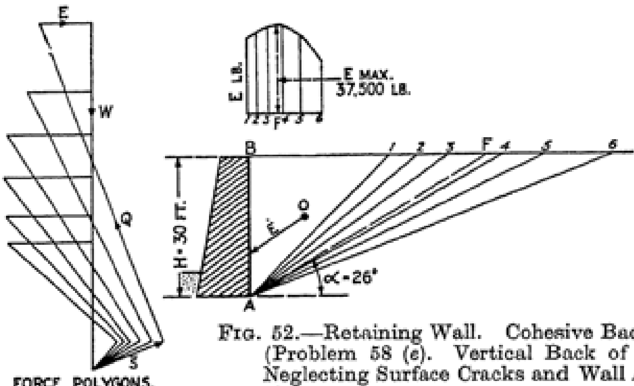


FIG. 52.—Retaining Wall. Cohesive Back-fill. (Problem 58 (*e*). Vertical Back of Wall. Neglecting Surface Cracks and Wall Adhesion.)

The foregoing results offer interesting comparisons in the effect on earth-pressure calculations when certain factors are neglected or conditions varied, and it will be appreciated that careful consideration must be given in each case to anticipate the conditions likely to exist when the construction works have been completed.

Nomograms for Earth Pressures—Cohesive Soils

The authors have constructed nomograms for the rapid estimation of both active earth pressure and passive earth resistance for

cohesive soils, and these are published in Figs. 53 and 54. The nomograms are based on formulæ given in a paper entitled "Pressure and Resistance of Clays" presented to the Institution

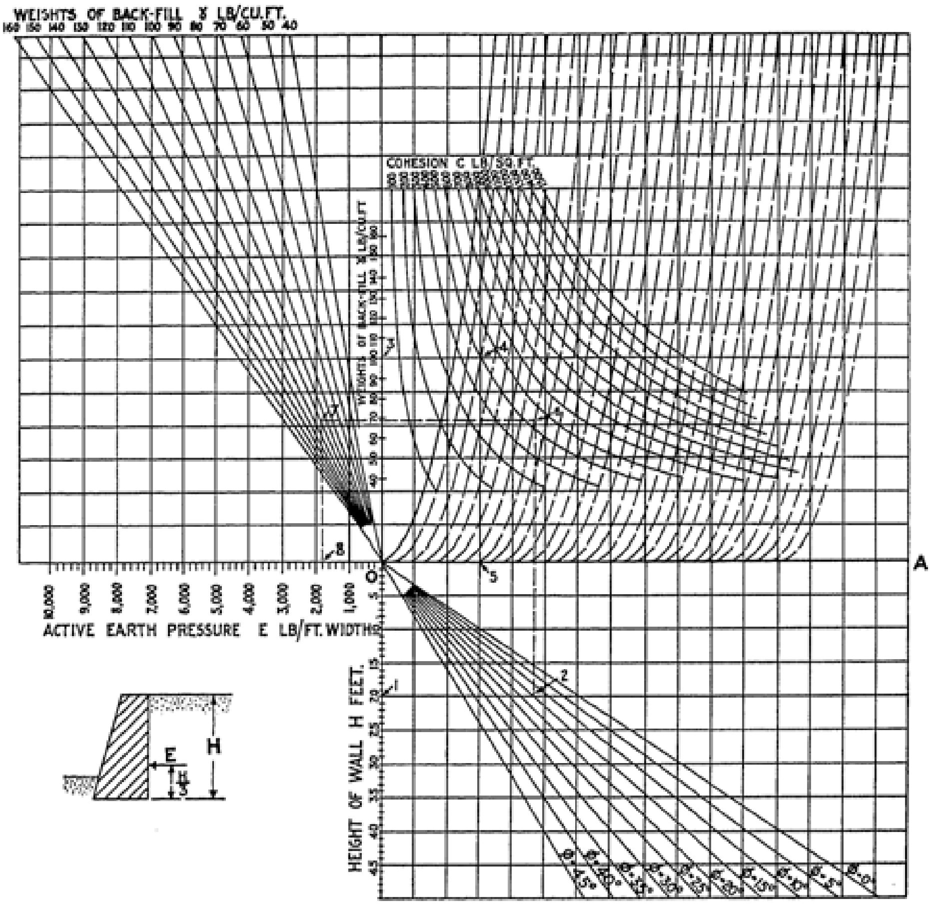


Fig. 53.—Nomogram for Active Earth Pressure, Cohesive Soils.

of Civil Engineers by Mr. A. L. Bell (Vol. 199, 1914) for walls with a vertical back and no surcharge of the back-fill.

The general formula for the active earth pressure for cohesive soil is as follows :—

$$E = \frac{1}{2}\left[H - \frac{2c}{\gamma}\cot\left(45^\circ - \frac{\phi}{2}\right)\right]$$
$$\left[\gamma H \tan^2\left(45^\circ - \frac{\phi}{2}\right) - 2c\tan\left(45^\circ - \frac{\phi}{2}\right)\right] \quad . \quad (32)$$

and this equation has been transformed to

$$E = \frac{\gamma}{2}\left[H\tan\left(45^\circ - \frac{\phi}{2}\right) - \frac{2c}{\gamma}\right]^2$$

for the purpose of constructing the nomogram. The earth pressure E given by this formula is the horizontal component, and neither wall friction nor adhesion is taken into account. The solution by trial polygons of forces is more accurate, but the nomograms are useful for rapid approximations.

As a comparison, the solution of Problem 58 (*e*) by means of equation (32) gives a horizontal earth pressure of 32,700 lb. per lin. ft. of wall, whereas the result obtained from trial polygons of forces to ascertain the critical slip plane is 33,400 lb. per lin. ft. of wall. These two results may be regarded as comparing satisfactory, as the difference is only 2 per cent.

Problem 59. A retaining wall is to be constructed to retain a clay back-fill 20 ft. high with a cohesion of 500 lb. per sq. ft. and a unit weight of soil of 100 lb. per cu. ft. The angle of internal friction is 15°. Ascertain the magnitude of the active earth pressure from the nomogram in Fig. 53.

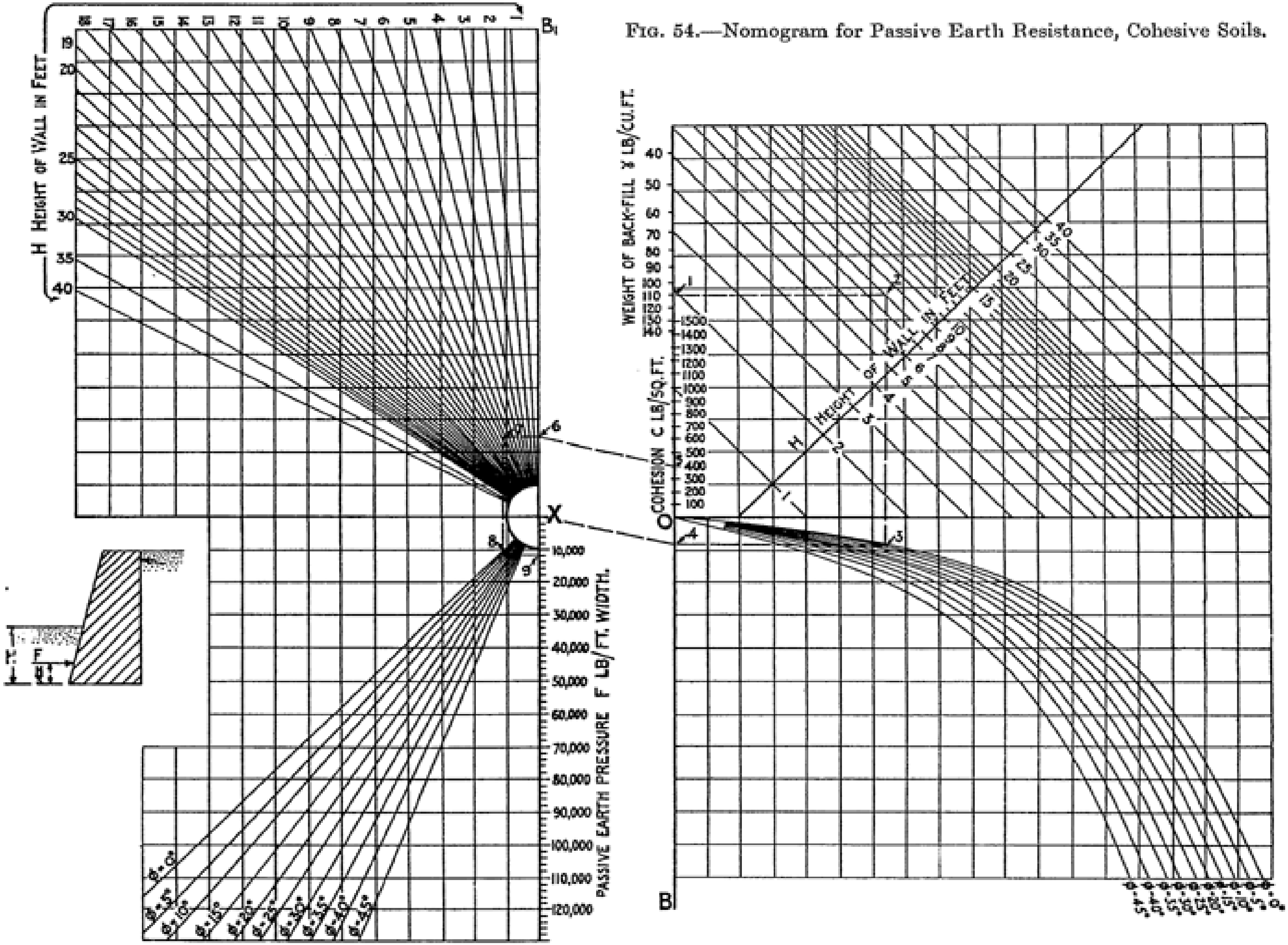
With reference to Fig. 53, from the scale for wall heights at 20 ft. at point (1) draw a horizontal to one of the diagonal lines at point (2) corresponding to $\phi = 15°$, and erect a perpendicular. From the point (3) at 100 lb. per cu. ft. on the scale of weights of back-fill draw a horizontal to cut the appropriate curve for cohesion amounting to 500 lb. per cu. ft. at the point (4) and drop a perpendicular to the line OA at point (5). From (5) follow the curve until the vertical from the point (2) is intersected at (6).

From the point (6) draw a horizontal to the diagonals for the weights of back-fill at (7) for 100 lb. per cu. ft. From the point (7) drop a perpendicular on to the earth-pressure scale at the point (8), reading the value of the horizontal component at 1,800 lb. per lin. ft. of wall acting at one-third the height of the wall.

The general formula for passive earth resistance of cohesive soils is as follows :—

$$F = \frac{H}{2}\left[\gamma H \tan^2\left(45^\circ + \frac{\phi}{2}\right) + 4c\tan\left(45^\circ + \frac{\phi}{2}\right)\right] \quad . \quad (33)$$

FIG. 54.—Nomogram for Passive Earth Resistance, Cohesive Soils.



and this equation has been transformed to

$$F = H \tan\left(45^\circ + \frac{\phi}{2}\right)\left[\frac{\gamma H}{2}\tan\left(45^\circ + \frac{\phi}{2}\right) + 2c\right]$$

for the purpose of constructing the nomogram. The earth resistance F given by this formula is the horizontal component, and both wall friction and adhesion is neglected.

Problem 60. An abutment wall is founded 8 ft. below ground level in a clay soil of weight 110 lb. per cu. ft., cohesion of 400 lb. per sq. ft. and angle of internal friction of 5°. Ascertain the magnitude of the passive earth resistance from the nomogram in Fig. 54.

With reference to Fig. 54, from the point (1) representing 110 lb. per cu. ft. on the scale for weights of soil, draw a horizontal line to intersect at point (2) the appropriate diagonal line for H = 8 ft. (depth of foundation), and from this point a vertical line is traced to cut the curve drawn for $\phi = 5^\circ$ at the point (3).

From (3) a horizontal line is drawn to a point (4) on the vertical axis OB, and this point at (4) is joined to the origin at X.

From a further point (5) representing the value of cohesion at 400 lb. per sq. ft. on the vertical scale, a line is drawn parallel to (4)X to intercept the vertical axis XB_1 at the point (6).

From the point (6) a horizontal is drawn to cut the straight line graphs for H = 8 ft. at the point (7). A vertical is drawn from (7) to cut the straight line graphs for $\phi = 5^\circ$ at point (8), and finally a horizontal is drawn to cut the scale for passive earth resistances at (9), where the value of F = 12,000 lb. per lin. ft. of abutment is read.

The full passive earth resistance may not be fully mobilised, and the result obtained from the nomogram can be suitably reduced by applying a factor of safety between 1·5 and 2.

Retaining Walls—Base Resistance

In addition to the calculation of earth pressures and resistances and the subsequent design of the retaining wall, it is necessary to investigate the resistance of the base of the wall against sliding and the possibility of failure by circular slip of the retaining wall and back-fill combined.

From inspection of Fig. 55 it will be realised that for stability the active earth pressure, E, will be balanced by the combined forces of the passive earth resistance, F, and the base resistance against sliding, Q_H.

$$E = F + Q_H \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

A factor of safety of 1·5 should be introduced in calculations for the resistance of the base against sliding.

When considering cohesionless soils,

$$Q_H = Q_V \times \tan \delta \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where Q_V is the total vertical load on the soil below the base, and δ is the angle of friction between the wall and the soil.

The angle of wall friction, δ, cannot exceed the angle of internal friction of the soil, and with materials such as sand or gravel the value is likely to be between 17° and 25°, dependent on the sharpness of the grains and the degree of compaction. Generally, a value of 0·5 to 0·75 times the value of ϕ may be adopted.

With cohesive soils the shear resistance is an important factor, and for stability against sliding,

$$Q_H = s \times b \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

where s is the shear resistance of the soil, and b is the breadth of the wall foundation.

The construction of the wall foundation to an angle, as indicated by the dash line XY in Fig. 55, will assist the resistance of the

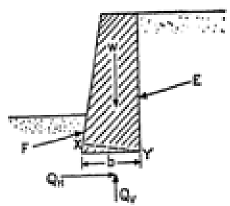


FIG. 55.—Retaining Wall. Base Resistance.

base and will help to distribute the pressure more evenly upon the soil. Extensive variation in the distribution of pressure on the foundation strata may cause more rapid consolidation of the clay, resulting in uneven settlement and tilting of the wall to an extent which may cause failure.

Problem 61. A retaining wall to be constructed in a cohesionless soil is calculated to have an active earth pressure of 7,500 lb. per lin. ft. exerted upon the back of the wall. The passive earth resistance is 4,000 lb. per lin. ft. and the total weight of the wall is 15,000 lb. per lin. ft. Assume the full value of the angle of wall friction of 26½° between the wall foundation and the soil occurs. What is the factor of safety of the wall against sliding?

Factor of safety, from equations (34) and (35)

$$= \frac{F + Q_H}{E} = \frac{F + (W \times \tan \delta)}{E} = \frac{4000 + (15000 \times 0{\cdot}5)}{7500} = 1{\cdot}53$$

This factor of safety may be regarded as satisfactory.

Problem 62. A retaining wall is to be constructed in a cohesive soil of shear resistance 500 lb. per sq. ft., and it is calculated that there will be an active earth pressure of 15,000 lb. per lin. ft. and a passive resistance of 20,000 lb. per lin. ft. What is the breadth of base required to give a factor of safety of 1·5 against sliding?

Factor of safety, from equations (34) and (36)

$$= \frac{F + Q_H}{E} = \frac{F + (s \times b)}{E} = 1{\cdot}5 = \frac{20000 + (500 \times b)}{15000}$$

Width of base required,

$$b = \frac{(1{\cdot}5 \times 15000) - 20000}{500} = 5'\ 0''$$

CHAPTER VIII

STABILITY OF RETAINING WALLS, SHEET PILING AND TRENCH TIMBERING

Retaining Walls—Circular Slip Conditions

Chapters VI and VII dealt with the methods for calculating earth pressures enabling the subsequent design of retaining walls to be carried out. When considering the construction of a retaining wall in a cohesive soil, however, it is essential to investigate the overall stability against circular slip. It is possible for rotational movement to occur in a slope and the arc of failure to be situated

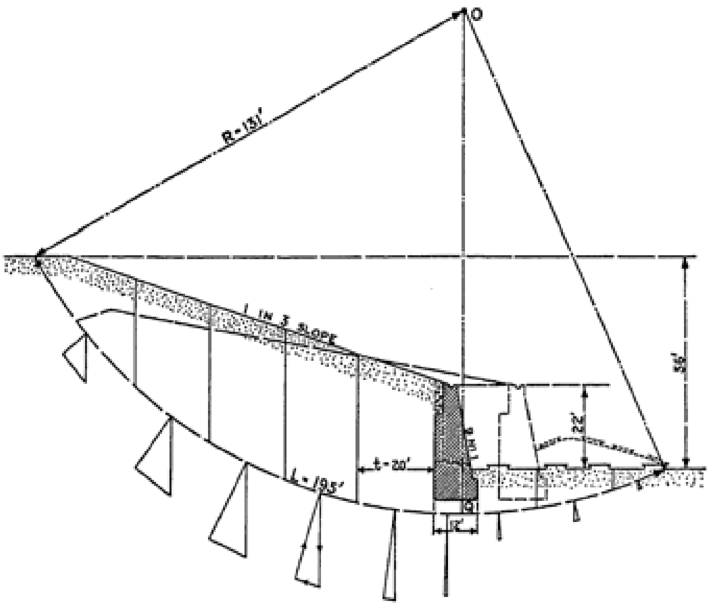


FIG. 56(a).—Overall Stability of Retaining Wall. (Problem 63.)

below the foundations of the retaining wall. In such a case it will be evident that any alteration in detail of the wall design, even if the thickness were doubled, would not increase the overall stability. The following Problems 63 and 64 will outline the method of investigating stability under these conditions.

Problem 63. The conditions which existed in a retaining wall failure near Wembley Hill station are indicated in Fig. 56(a). The

height of the wall was 22 ft. above formation level with a face batter of 1 in 6 and a thickness of 12 ft. at the base. The total depth of the cutting was 56 ft., and the back-fill had a slope of 1 in 3 from the top of the wall. The soil retained consisted of a plastic type clay with a cohesion of 600 lb. per sq. ft., no apparent angle of internal friction and a weight of 110 lb. per cu. ft. The weight of the concrete in the wall may be taken as 120 lb. per cu. ft.

Investigate the factor of safety against circular slip conditions.

The method employed is similar to that already described in Chapter III for ascertaining the critical slip-circle for earth-slopes. The centres of trial slip-circles lie on a perpendicular above the face of the retaining wall, as indicated by OQ in Fig. 56(a). The method of slices is employed for a number of trial circles, and a factor of safety is calculated in each case from equation (22) as follows :—

$$\text{Factor of safety,} \quad F = \frac{L \times c + (\tan \phi \Sigma N)}{\Sigma T}$$

but if the angle of internal friction is zero then

$$F = \frac{L \times c}{\Sigma T}$$

The values of the factor of safety are plotted on the line OQ as base opposite the appropriate centres, and these points are joined to form a smooth curve. This curve gives a minimum value for the factor of safety which enables the centre of the critical circle O to be determined.

Tabular results are as follows :—

Slice No.	Tangential component, T.	Other data.	Factor of safety, F.
1	40,040 lb.	$L = 195$ ft.	$\frac{L \times c}{\Sigma T} =$
2	42,900 ,,	$R = 131$ ft.	
3	37,400 ,,	$c = 600$ lb. per sq. ft.	$\frac{195 \times 600}{148,710} = 0{\cdot}79.$
4	24,200 ,,	$\phi =$ zero.	
5	11,000 ,,	$\gamma = 110$ lb. per cu. ft. (clay).	
6	1,200 ,,		
7	− 2,200 ,,	$w = 120$ lb. per cu. ft. (concrete).	
8	− 4,400 ,,		
9	− 1,430 ,,		

$\Sigma T = 148,710$ lb.

From the above analysis it is apparent that the slope and retaining wall are unstable and a circular slip can occur.

When failure occurred, the retaining wall moved forward a distance of 18 ft. in half an hour. The top of the slope subsided

16 ft. and the railway formation was lifted a height of 9 ft. in front of the wall, as indicated in Fig. 56(a).

Problem 64. A retaining wall with a level back-fill is 30 ft. high above cess level. The base of the wall is founded 10 ft. below cess level and has a thickness of 13 ft. as shown in Fig. 56(b). The back-fill consists of clay with a weight of 110 lb. per cu. ft.,

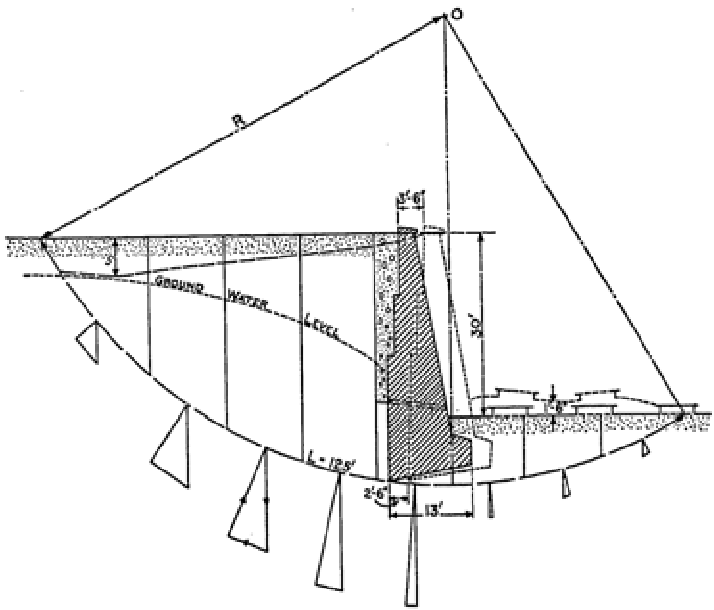


FIG. 56(b).—Overall Stability of Retaining Wall. (Problem 64.)

cohesive strength of 500 lb. per sq. ft., and the value for the angle of internal friction is small, and may be neglected. The weight of the masonry in the wall is 120 lb. per cu. ft. Investigate the overall stability of the wall against circular slip failure.

A vertical line is drawn through the face of the wall and, as described in the previous problem, a number of centres are taken on this line, and slip-circles are drawn for possible arcs of failure. Opposite each centre the appropriate factor of safety is plotted from calculations based on equation (22). These points representing the factors of safety are joined to form a curve, and the minimum value identified the centre of the critical circle O. as shown in Fig. 56(b).

Tabular results are as follows :—

Slice No.	Tangential component, T.	Other data.	Factor of safety, F.
1	19,250 lb.	L = 125 ft.	$\frac{L \times c}{\Sigma T} =$
2	21,312 ,,	c = 500 lb. per sq. ft.	
3	19,250 ,,	ϕ = zero.	$\frac{125 \times 500}{70,252} = 0{\cdot}89.$
4	13,062 ,,	γ = 110 lb. per cu. ft. (clay).	
5	4,500 ,,		
6	− 1,375 ,,	w = 120 lb. per cu. ft. (masonry).	
7	− 3,437 ,,		
8	− 2,310 ,,		

$\Sigma T = 70,252$ lb.

From this result it is apparent that the wall is unstable. Failure occurred by a horizontal forward movement of 3 ft. 6 ins. at the coping and 2 ft. 6 ins. at the base of the wall. The back-fill subsided 5 ft. and the upheaval of the railway tracks amounted to 18 ins.

Sheet Piling

In connection with the design of sheet piling several methods are available for calculating the penetration required to attain stability with a satisfactory factor of safety. It is considered that the most simple approach to sheet piling design is by a method attributed to Blum (1930–31) and the procedure necessary for solving sheet piling problems for both "free earth" and "fixed earth" conditions will be briefly described.

Method for Design (Blum).

The forces acting upon the sheet piling are :—

(1) the tie-bar or anchor pull, T,

(2) the active earth pressure of the back-fill, P, and

(3) the passive earth resistance in front of the foot of the piling, F.

The data and procedure for estimating earth pressures and resistances, together with the use of the nomograms as published for retaining walls in Chapters VI and VII, may be used for sheet piling as conditions are similar. The influence of superimposed loads, both concentrated and distributed, are dealt with in exactly the same manner as outlined in Chapter VI. Furthermore, the effect on earth pressure due to surcharge, varying types of back-fill and submerged conditions are calculated as indicated in Problems 50 to 54.

(a) *"Free Earth" Support.*

The diagram of the distribution of pressure will be as indicated in Fig. 57 for free end conditions, and the yield of the sheet piling is indicated by dotted lines. In the construction of the distribution of pressure diagram UV is drawn parallel to XY for passive earth pressure.

The length of the piling is divided into a number of equal sections, lettered A to K, and a vector diagram is constructed by setting out horizontally the forces a to k acting in each section. Selecting a polar distance, lines are drawn from each point representing the forces a to k on the vector line to the pole O.

A funicular polygon of bending moment is then constructed by drawing alongside the sheet piling, lines parallel to those of the vector diagram within the limits of the horizontal lines for each

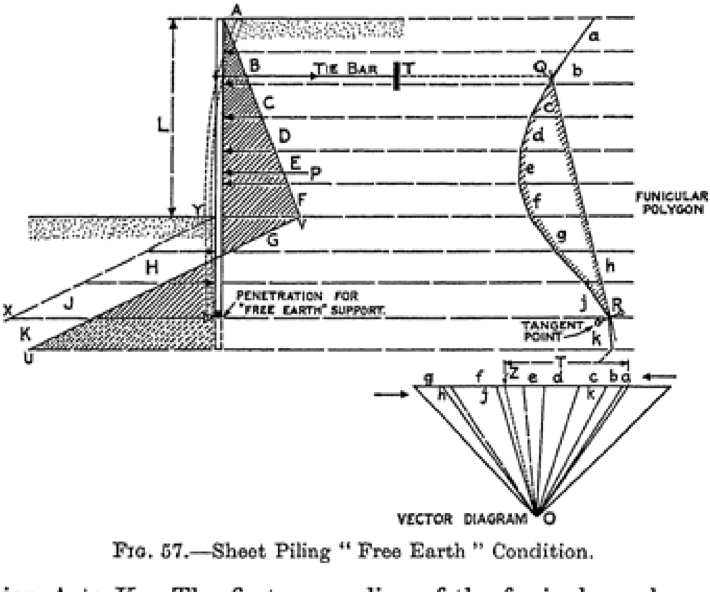


Fig. 57.—Sheet Piling "Free Earth" Condition.

section A to K. The first upper line of the funicular polygon is produced to cut the horizontal line corresponding to the tie-bar pull at a point Q.

From the point Q a base line is drawn to tangent the lower portion of the funicular polygon at a point R. The line QR is the base line for "free earth" support, and the projection horizontally of the point R on the pile diagram will indicate the depth at which the pile may be driven to provide "free-earth" support.

To calculate the tie-bar pull, draw a line from O to the vector line at Z parallel to the base line QR, and the force a to Z will be the force in the anchor bar.

(b) *" Fixed Earth " Support.*

The diagram of the distribution of pressure is indicated in Fig. 58 for fixed-end conditions, and the deflection of the piling is shown by dotted lines. It is possible that slight deflection may occur at the toe of the pile, but it is so small that it may be neglected.

The procedure is followed as described above for " free-earth " support, and the vector diagram and funicular polygon constructed in a similar manner.

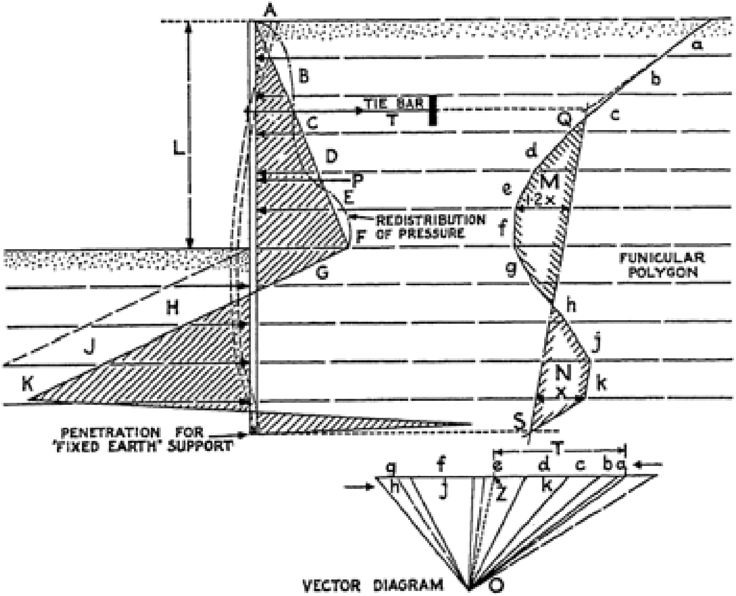


FIG. 58.—Sheet Piling " Fixed Earth " Condition.

From the point Q a base line is drawn so that the area M multiplied by the distance of its centre of gravity from Q is equal to the area N multiplied by the distance of its centre of gravity from Q. This may be accomplished quickly by making the maximum horizontal ordinate of M equal to 1·2 times the maximum horizontal ordinate of N. The base line from Q will cut the funicular polygon at S, and the horizontal projection of S on the pile diagram indicates the depth of penetration required to attain " fixed-earth " conditions.

To ensure a satisfactory penetration of the piling, including an allowance for slight deflection of the toe which has been neglected,

it is advisable to increase the depth of penetration found by the above method by 15 per cent.

The distribution of pressure on the back of the piling is not uniform, and is of the form indicated by a chain-dotted line in Fig. 58. This assumed redistribution of pressure reduces the maximum bending moment up to 25 per cent., but increases the anchor tie pull by 10 per cent.

Problem 65. Sheet piling is to be driven to act as a retaining wall 20 ft. high. The soil is of a sandy type, with an angle of repose of 30°, weight of 110 lb. per cu. ft., and the surface of the back-fill is level with the top of the piling. It is proposed to provide an anchor tie at 5 ft. below the top of the piling. What penetration of the piling is necessary to provide (*a*) " free-earth " support and (*b*) " fixed-earth " support?

From nomograms in Figs. 44 and 45 (Chapter VI) the active earth pressure P is 7,000 lb. per ft. run, and the practical passive earth resistance F is 11,500 lb. per ft. run. This data will enable the earth-pressure diagram to be constructed as shown in Fig. 59. The length of the sheet piling is divided into a number of equal sections of 4 ft., and lettered A to K.

The vector diagram is drawn by setting out horizontally the forces in each section a to k and joining the points thus obtained to the pole O.

The funicular polygon is then constructed by drawing alongside the pile diagram lines parallel to those of the vector diagram. The initial line, a, of the polygon is produced to cut the line relating to the tie-bar pull, T, at the point Q.

(a) " *Free-Earth* " *Support.* To ascertain the depth of penetration of the piling under this condition, a base line is drawn from Q to tangent the lower portion of the funicular polygon at R. The point R is projected horizontally on to the pile diagram, which gives the depth of penetration necessary as 12 ft.

A line OZ is drawn on the vector diagram parallel to the base line QR, and the anchor tie-pull, T_1, is ascertained from this diagram to be 2·03 tons. Due to arching action of the back-fill, this pull will be increased by 10 per cent. to 2·25 tons, and the anchor should be designed for this working load per ft. run of piling.

(b) " *Fixed-Earth* " *Support.* A base line is drawn from Q to cut the lower portion of the funicular polygon at S, so that the greatest horizontal ordinate of the area M is 1·2 the greatest horizontal ordinate of N. In this case the ordinate at 16 ft. deep is 11·4, which is 1·2 times that of 9·5 occurring at a depth of 32 ft.

The horizontal projection of the point S on the pile diagram gives a theoretical depth of penetration of 18·5 ft., neglecting toe deflection of the pile. This value should be increased by 15 per

cent., and a satisfactory penetration would be 21·25 ft. in this instance.

The line OY is drawn parallel to the base line QS, as shown in Fig. 59, and this gives a tie-bar pull on the vector line of 1·7 tons per ft. run, which should be increased to a working load of 1·87 tons per ft. run, to allow for the arching effect of the back-fill.

Overall Stability of Sheet Piling.

Earlier in this chapter reference was made to the necessity of investigating the overall stability of retaining walls in clays against

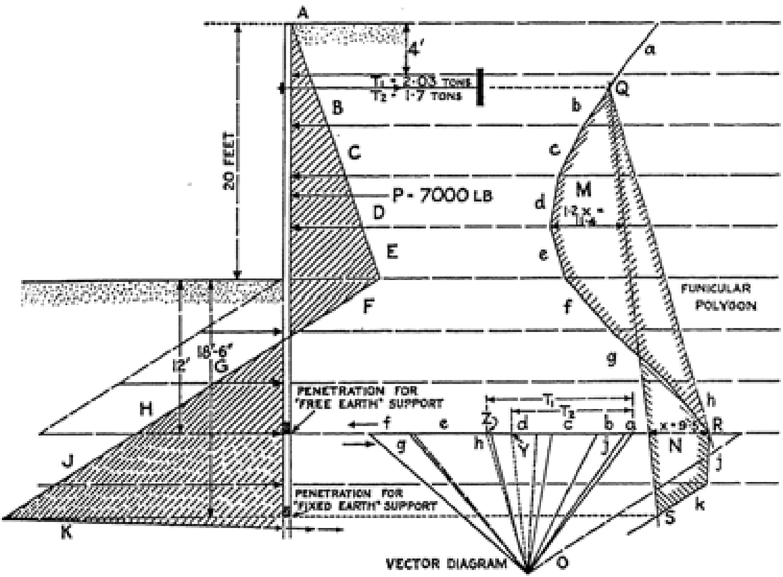


Fig. 59.—Sheet Piling. (Problem 65.)

failure by circular slip, and when considering sheet piling similar calculations must be made. Calculations and procedure are identical with those outlined in Problems 63 and 64. A number of centres for potential slip circles are chosen on a vertical line through the face of the sheet piling. The method of slices is employed for each circle, and a factor of safety calculated in each case from equation (22). The various values for the factor of safety are plotted opposite each centre, and these points joined to form a smooth curve, thus enabling the minimum value to be obtained and the critical slip circle established.

Critical Height of Sheet Piling.

Sheet piling of height H, in clay, is subjected to an active earth pressure $= \gamma H - 2s$ and a passive earth resistance $= 2s$, where s is the shear strength of the clay and γ is the unit weight of the soil.

Equilibrium will just be possible when $\gamma H - 2s = 2s$.

Therefore, the critical height of the sheet piling is

$$H = \frac{4s}{\gamma}$$

It will be evident that equilibrium cannot be attained if $\frac{s}{\gamma H}$ is less than 0·25, unless the piling is driven to a deeper and stronger stratum of sand or clay. (It should be noted that the ratio $\frac{s}{\gamma H}$ is similar to that used in Taylor's curves in Chapter III for the "stability number.")

Generally it is advisable to check the overall stability and critical height of a proposal to adopt sheet piling before the actual design is commenced as in Problem 65.

Problem 66. A sheet pile wall 22 ft. high is required to retain a clay soil of shear strength of 420 lb. per sq. ft. Water level will exist to a height of 12 ft. in front of the wall. The specific gravity of the soil is 2·25, and the void ratio is 50 per cent. Investigate the general equilibrium of the sheet piling.

The submerged density of the clay retained (from equation 8)

$$= \frac{\rho - 1}{1 + e}\gamma_w = \left(\frac{2{\cdot}25 - 1}{1 + 0{\cdot}5}\right) \times 62{\cdot}5 = 52{\cdot}1 \text{ lb. per cu. ft.}$$

The clay is likely to be saturated, and the bulk density above water level (from equation 7)

$$= \frac{\rho + e}{1 + e}\gamma_w \times \left(\frac{2{\cdot}25 + 0{\cdot}5}{1 + 0{\cdot}5}\right) \times 62{\cdot}5 = 107{\cdot}9 \text{ lb. per cu. ft.}$$

Average density of filling

$$= \frac{(10 \times 107{\cdot}9) + (12 \times 52{\cdot}1)}{22} = 77{\cdot}5 \text{ lb. per cu. ft.}$$

Therefore the ratio $\frac{s}{\gamma H} = \frac{420}{77{\cdot}5 \times 22} = 0{\cdot}246$.

The stability number $\frac{s}{\gamma H}$ is less than 0·25, and it is evident that equilibrium of the sheet piling cannot be attained at any

depth of penetration unless the shear strength of the clay increases or the piling penetrates a stronger stratum of sand or gravel.

Timbering Excavations

During excavations which are being timbered there is an inward yield of the soil as the first set of struts are subjected to compressive stress. This yield increases when the second set is placed, and continues to do so as the excavations are taken deeper.

With reference to Fig. 60, it will be noted that the slip plane approximates to a circular type, and it will be appreciated that in the upper portion of the earth-wedge there is a greater volume

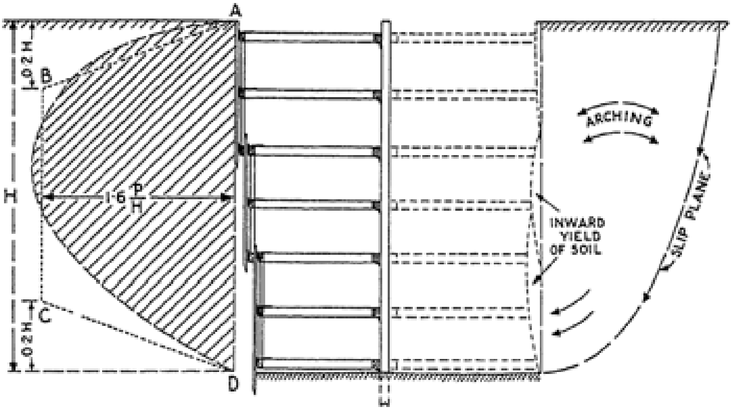


FIG. 60.—Timbering of Excavations. Distribution of Earth Pressure.

of soil to expand a small amount compared with the lower portion, which has a less volume to expand a great amount. Thus, in the upper portion "arching" of the soil particles occurs, and this has the effect of raising the centre of pressure.

Field observations in sandy clay (coarse sand with 25 per cent. clay) were made in Flatbush Avenue, New York City; in sandy gravel at the Siemen's Bau Union, Berlin; recently in clay at the Chicago Subway, by Terzaghi; and in London by the Building Research Station: and these investigations show that the distribution of pressure is parabolic, with the centre of pressure at approximately the half-point as indicated on the left in Fig. 60.

In design an enveloping trapezoid may be used as shown by ABCD in Fig. 60. The area of this figure will be in excess of that of the parabola, but this may be disregarded, as it errs on the side of safety. The active earth pressure may be calculated as already described for retaining walls and sheet piling, but the pressure

will be 1·28 times this value in excavations, and the point of application will be at half the depth of the excavation. The maximum pressure will be $1{\cdot}6 \times \frac{P}{H}$, which will extend from 0·2H to 0·8H.

The depth to which an excavation in clay may be taken without timbering and adopting a satisfactory factor of safety has been referred to in Chapter III, Problem 35, making use of Taylor's curves published in Figs. 16 and 17.

Problem 67. An excavation in sand is taken down to a depth of 28 ft. The soil has a weight of 105 lb. per cu. ft. and an angle of friction of 35°. Ascertain the magnitude of the pressure on the timber sheeting of the excavation and the point of application.

From the nomogram published as Fig. 44 for cohesionless soils the active earth pressure for a height of 28 ft., $\phi = 35°$ and weight of soil = 105 lb. per cu. ft. is ascertained to be approximately 10,000 lb. per ft. run.

Therefore, the total load on the timbering will be

$$10{,}000 \times 1{\cdot}28 \text{ lb.} = 12{,}800 \text{ lb.}$$

This load acts at the mid-point of the depth of the excavation or at 14 ft. below ground level.

The maximum pressure will be

$$\frac{12{,}800}{0{\cdot}8\text{ H}} = \frac{12{,}800}{0{\cdot}8 \times 28} = 571 \text{ lb. per sq. ft.}$$

and this will extend from 5·6 ft. to 22·4 ft. below ground level.

Note on "sand arching" in connection with retaining walls, sheet piling, etc.

"Sand arching" can be induced in the back-fill to a retaining wall if the wall moves forward horizontally a small amount, which may be of the order of 0·1 per cent. of the height—*i.e.*, a movement of 0·25 in. in a wall of 20 ft. height. The earth pressure is redistributed and the pressure in the upper portion of the back-fill is increased, whilst that in the lower portion is decreased, but the value of the total pressure does not change.

The point of application of the resultant total pressure is higher than the one-third point, and lies between 0·45H and 0·55H above the ground level. It must be realised, however, that "arching" cannot take place before the retaining wall, sheet piling or trench timbering translates or deflects, and there cannot be "arching" in a soil mass in a state of equilibrium. "Arching" occurs in similar circumstances in clays, but in this type of soil it takes place to a much smaller degree.

CHAPTER IX

FOUNDATIONS—BEARING CAPACITY

The design of a foundation for a structure is governed by two main considerations :—

1. The soil must be safe against failure by shear, which may cause plastic flow under the structure.
2. The structure must be safe against excessive settlement or movement due to consolidation of the soil under the foundation.

For all civil engineering structures it is important to have soil tests taken and the shearing value established for the soil in each strata occurring below the foundation of the structure to a depth of at least one-and-a-half times the breadth of the foundation. The safe bearing power of a soil to carry light buildings in a town are usually determined from experience supported by information relating to foundations of structures already existing.

During the construction of new railways or roads it is not unusual for the bearing capacity of the soil in the excavations for bridges or other structures to be determined simply by visual inspection. In a measure this may be satisfactory if the Resident Engineer has experience and knowledge of the soils occurring in the locality, but often undue settlement causing fractures in the structure indicate that the soil is less resistant than it appeared to be on first inspection. In fact, complete failure of a structure might occur if the ultimate bearing capacity of the soil were exceeded.

It cannot be too strongly recommended that borings and shear tests should be made in all cases where structures of any importance are to be built.

Pressure Distribution—Vertical Pressure

The distribution of normal vertical stress is needed for the solution of settlement problems, and a diagram with isolines joining points of equal pressure gives rise to the well-known " bulb-of-pressure " diagram, as indicated in Fig. 62. It will be apparent from these diagrams that the larger the loaded area, the deeper is its influence and, furthermore, if a number of loaded footings are closely spaced, the effects of each footing merge into one large pressure-bulb.

Reference has already been made in Chapter VI (Equation 30) to Boussinesq's formula for computing the intensity of vertical

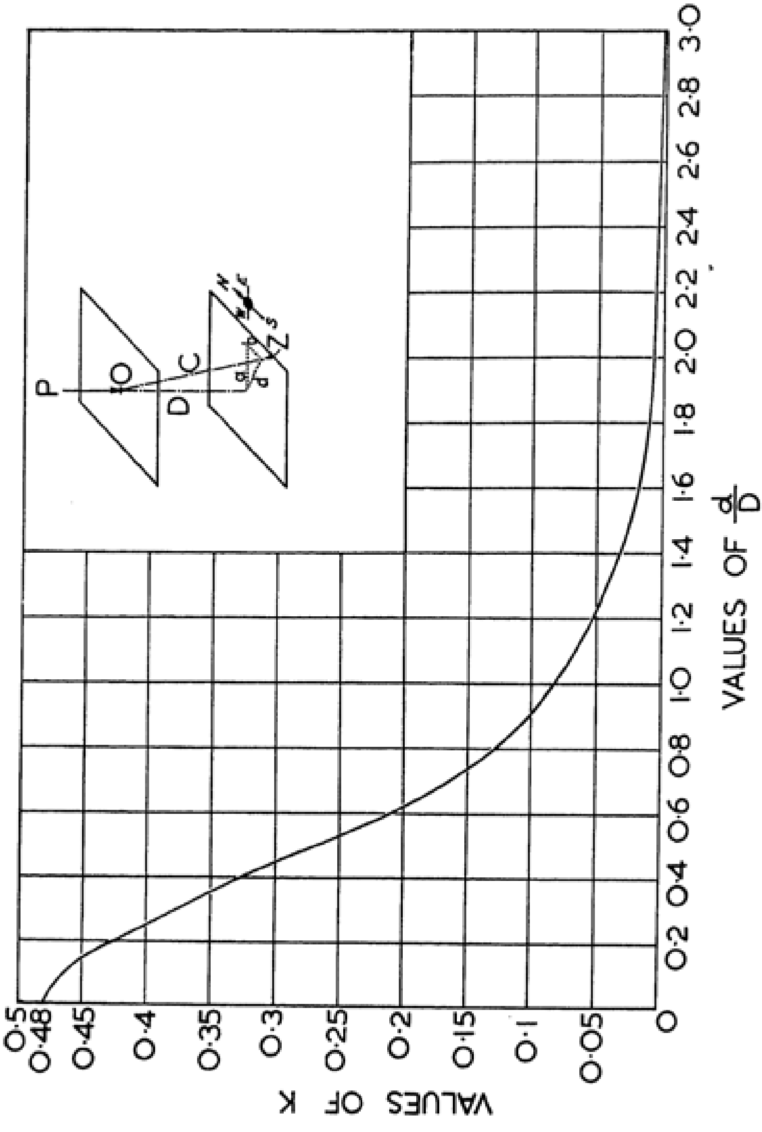


FIG. 61.—Boussinesq Formula Values of K.

pressure at any depth, and this feature will now be studied in more detail. With reference to Fig. 61, assume a point load, P, acting at a point O on a surface; it is desired to ascertain the

intensity of vertical pressure at a point, Z, situated at a depth, D, and a distance, d, away horizontally.

The intensity of pressure at Z will be

$$q = \frac{3PD^3}{2\pi C^5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where $C = \sqrt{d^2 + D^2}$ and $d = \sqrt{a^2 + b^2}$

if d denotes the distance horizontally from the vertical through the point of application of the load, P, at the depth, D;
a denotes the distance east or west of this vertical at depth, D;
b denotes the distance north or south of this vertical at depth, D.

This formula may be written as

$$q = \frac{KP}{D^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where K is a constant which may be determined from the graph given in Fig. 61 obtained from the value of the ratio $\frac{d}{D}$.

If a number of concentrated loads are considered, then the stress intensities at a point from the various loads may be added. Thus, when a uniformly distributed load is considered, the loaded area may be divided into a number of small areas and the load on each area regarded as a concentrated load.

Problem 68. Assume a load of 20 tons as a concentrated load at ground level. Determine the vertical pressure at a point 25 ft. below the surface and 10 ft. away horizontally.

The ratio $$\frac{d}{D} = \frac{10}{25} = 0{\cdot}4$$

The value of K from Fig. 61 is 0·32.
Hence the intensity of pressure at the point referred to is

$$q = \frac{KP}{D^2} = \frac{0{\cdot}32 \times 20}{25 \times 25} = 0{\cdot}01 \text{ tons per sq. ft.}$$

Problem 69. A raft foundation carries a uniformly distributed load of 2·5 tons per sq. ft. The raft is 20 ft. wide and 50 ft. long, and may be regarded as non-rigid. Calculate the intensity of vertical pressure at a point 15 ft. deep below the centre of the raft.

The area of the raft can be conveniently divided up into twelve areas each 10 ft. by 8 ft. 4 ins. The load on each of these areas

will be 2·5 tons by 10 ft. by 8 ft. 4 ins. = 208·3 tons. This load may be regarded as a concentrated load acting at the centre of each of the twelve areas. The whole raft consists of four symmetrical portions of three of these areas, and it will be sufficient to calculate these three areas and multiply by four.

The distances, d, from the centres of these areas are as follows :—

$$d_1 = \sqrt{5^2 + 4{\cdot}16^2} = 6{\cdot}5 \text{ ft.}$$
$$d_2 = \sqrt{5^2 + 12{\cdot}5^2} = 13{\cdot}46 \text{ ft.}$$
$$d_3 = \sqrt{5^2 + 20{\cdot}83^2} = 21{\cdot}42 \text{ ft.}$$

Hence the ratios $\frac{d}{D}$ and the constant K are :—

$$\left.\begin{array}{ll} \frac{d_1}{D} = \frac{6{\cdot}5}{15} = 0{\cdot}433 & \therefore K_1 = 0{\cdot}32 \\ \frac{d_2}{D} = \frac{13{\cdot}46}{15} = 0{\cdot}897 & \therefore K_2 = 0{\cdot}10 \\ \frac{d_3}{D} = \frac{21{\cdot}42}{15} = 1{\cdot}428 & \therefore K_3 = 0{\cdot}03 \end{array}\right\} \text{Total for } K = 0{\cdot}45$$

The intensity of vertical pressure,

$$q = \frac{KP}{D^2} = \frac{4 \times 0{\cdot}45 \times 208{\cdot}3}{15 \times 15} = 1{\cdot}65 \text{ tons per sq. ft.}$$

It will be evident that at the depth of 15 ft. below the raft foundation this intensity of pressure will be the maximum value at this depth, as the point chosen is directly under the centre of the raft.

Problem 70. A monument weighing 1,000 tons can be considered as a concentrated load. Calculate the vertical pressure under the structure at a depth of 15 ft. in a clay soil.

The ratio $\frac{d}{D}$ is zero and from Fig. 61, K = 0·48.

Hence the intensity of vertical pressure,

$$q = \frac{KP}{D^2} = \frac{0{\cdot}48 \times 1000}{15 \times 15} = 2{\cdot}13 \text{ tons per sq. ft.}$$

Problem 71. Two square buildings 60 ft. by 60 ft. are uniformly loaded with 3·5 tons and 2·5 tons per sq. ft. at the base respectively. A roadway 10 ft. wide separates the two buildings. Calculate the intensity of vertical pressure under the centre of the building loaded to 2·5 tons per sq. ft. at a depth of 20 ft.

Divide the foundation of the lighter building into 16 areas each 15 ft. square. The total load per square will be 562 tons.

The distances to the centres of these squares from the centre of the foundation will be as follows :—

Four distances $d_1 = \sqrt{7{\cdot}5^2 + 7{\cdot}5^2} = 10{\cdot}65'$

$$\frac{d_1}{D} = 0{\cdot}53 \qquad \therefore\ K_1 = 0{\cdot}26$$

Eight distances $d_2 = \sqrt{7{\cdot}5^2 + 22{\cdot}5^2} = 23{\cdot}7'$

$$\frac{d_2}{D} = 1{\cdot}18 \qquad \therefore\ K_2 = 0{\cdot}055$$

Four distances $d_3 = \sqrt{22{\cdot}5^2 + 22{\cdot}5^2} = 31{\cdot}9'$

$$\frac{d_3}{D} = 1{\cdot}59 \qquad \therefore\ K_3 = 0{\cdot}02$$

$$\Sigma K = 1{\cdot}56$$

Intensity of pressure due to lighter building

$$= q_1 = \frac{KP}{D^2} = \frac{1{\cdot}56 \times 562}{20 \times 20} = 2{\cdot}19 \text{ tons per sq. ft.}$$

Divide the foundation of the heavier building into a similar number of areas. The total load per square will be 787·5 tons.

The distances of the centres of these squares from the centre of the foundation will be as in Table 7 :—

TABLE 7.

(*Problem* 71.)

Two distances $d_4 = \sqrt{47{\cdot}5^2 + 7{\cdot}5^2} = 48{\cdot}0$	$\frac{d_4}{D} = 2{\cdot}4$	$K_4 = 0{\cdot}0025$
,, ,, $d_5 = \sqrt{47{\cdot}5^2 + 22{\cdot}5^2} = 52{\cdot}5$	$\frac{d_5}{D} = 2{\cdot}7$	$K_5 = 0{\cdot}002$
,, ,, $d_6 = \sqrt{62{\cdot}5^2 + 7{\cdot}5^2} = 5{\cdot}61$	$\frac{d_6}{D} = 2{\cdot}8$	$K_6 = 0{\cdot}002$
,, ,, $d_7 = \sqrt{62{\cdot}5^2 + 22{\cdot}5^2} = 60{\cdot}0$	$\frac{d_7}{D} = 3{\cdot}0$	$K_7 = 0{\cdot}002$
,, ,, $d_8 = \sqrt{77{\cdot}5^2 + 7{\cdot}5^2} = 78{\cdot}0$	$\frac{d_8}{D} = 3{\cdot}9$	$K_8 = 0{\cdot}001$
,, ,, $d_9 = \sqrt{77{\cdot}5^2 + 22{\cdot}5^2} = 80{\cdot}7$	$\frac{d_9}{D} = 4{\cdot}0$	$K_9 = 0{\cdot}001$
,, ,, $d_{10} = \sqrt{92{\cdot}5^2 + 7{\cdot}5^2} = 93{\cdot}0$	$\frac{d_{10}}{D} = 4{\cdot}6$	$K_{10} = 0{\cdot}0005$
,, ,, $d_{11} = \sqrt{92{\cdot}5^2 + 22{\cdot}5^2} = 95{\cdot}0$	$\frac{d_{11}}{D} = 4{\cdot}7$	$K_{11} = 0{\cdot}0005$
		$\Sigma K = 0{\cdot}0115 \times 2$ $= 0{\cdot}023$

Intensity of pressure due to heavier building

$$= q_2 = \frac{KP}{D^2} = 2 \times \frac{0{\cdot}023 \times 787{\cdot}5}{20 \times 20} = 0{\cdot}091 \text{ ton per sq. ft.}$$

Hence total intensity of vertical pressure due to both buildings

$$= q_1 + q_2 = 2{\cdot}19 + 0{\cdot}091 = \underline{2{\cdot}28 \text{ tons per sq. ft.}}$$

Problem 72. Calculate the intensity of vertical pressure at a point 25 ft. below the corner of a raft foundation considered to be non-rigid. The raft is 20 ft. by 50 ft., and is loaded to 3 tons per sq. ft.

Divide the raft into four areas of 10 ft. by 25 ft., which will each have a total load of 750 tons. (For greater accuracy of results the raft may be divided into a larger number of areas.)

The distances of the centre of these areas from one corner are as follows :—

One distance $d_1 = \sqrt{12{\cdot}5^2 + 5^2} = 13{\cdot}47$

$$\frac{d_1}{D} = 0{\cdot}54 \qquad \therefore K_1 = 0{\cdot}24$$

Two distances $d_2 = \sqrt{37{\cdot}5^2 + 5^2} = 37{\cdot}83$

$$\frac{d_2}{D} = 1{\cdot}513 \qquad \therefore K_2 = 0{\cdot}025$$

One distance $d_3 = \sqrt{37{\cdot}5^2 + 37{\cdot}5^2} = 53{\cdot}03$

$$\frac{d_3}{D} = 2{\cdot}12 \qquad \therefore K_3 = 0{\cdot}005$$

$$\underline{\underline{\Sigma K = 0{\cdot}27}}$$

Hence the intensity of vertical pressure under one corner of the raft foundation

$$= q = \frac{\Sigma KP}{D^2} = \frac{0{\cdot}27 \times 750}{25 \times 25} = 0{\cdot}324 \text{ ton per sq. ft.}$$

Circular Loaded Areas

A circular loaded base on clay with a radius r, produces a

$$\text{pressure } q = \frac{p}{2\sqrt{1 - c^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

at a point cr from the centre, where p denotes the unit load carried by the base.

By this equation, the pressure equals 0·5p at the centre, where c is zero, and is infinite at the edges, where c is unity, but, as no structure is perfectly rigid, the pressure at the edges equals about 1·3p as found from experiments.

The intensity of pressure due to a non-rigid base may be calculated from the equations (37) and (38).

With cohesionless soils circular loaded areas tend to become overstressed at the centre, and with a rigid base the pressure at the middle becomes 1·6p. Experiments with loaded circular flexible bases on such soils indicate that the pressure at the centre increases to 3p, where p denotes the unit load.

Problem 73. A circular foundation 10 ft. in diameter is loaded to 3 tons per sq. ft. Estimate and compare the pressures directly under the base at the edge and at the centre if founded on a cohesionless soil for either flexible or rigid bases. What will be the pressures for a rigid base on a clay soil ?

From the foregoing paragraph on circular loaded areas :—

Cohesionless soil. Flexible base. (1) Pressure at centre = 3p = 9 tons per sq. ft.

(2) Pressure at edge = Nil.

From experiments $q = \frac{p}{4}$

= 0·75 ton per sq. ft.

Cohesionless soil. Rigid base. (3) Pressure at centre = 1·6p = 4·8 tons per sq. ft.

(4) Pressure at edge = Nil.

From experiments $q = \frac{p}{3}$

= 1 ton per sq. ft.

Cohesive soil. Rigid base. (5) Pressure at centre = 0·5p = 1·5 tons per sq. ft.

(6) Pressure at edge = 1·3d = 3·9 tons per sq. ft.

Problem 74. A circular masonry tower is 20 ft. in diameter, and the load on the foundation is 4 tons per sq. ft. Calculate the vertical pressure in a clay soil at a depth of 25 ft. under the centre of the tower if the base is (*a*) rigid and (*b*) flexible.

TABLE 8. (*Problem 74a.*)

Area.	Load on ring.	$\frac{d}{D}$ & K.	Stress at depth $= \frac{Kp}{D^2}$.
Outer ring . . .	$4{\cdot}55 \times \pi \times \frac{20^2 - 15^2}{4} = 629$ tons	$\frac{17{\cdot}5}{26} = 0{\cdot}7$, $K_1 = 0{\cdot}18$	$\frac{0{\cdot}18 \times 629}{25 \times 25} = 0{\cdot}18$ ton per sq. ft.
Second ring . . .	$2{\cdot}66 \times \pi \times \frac{15^2 - 10^2}{4} = 261$ tons	$\frac{12{\cdot}5}{25} = 0{\cdot}5$, $K_2 = 0{\cdot}27$	$\frac{0{\cdot}27 \times 261}{25 \times 25} = 0{\cdot}11$ ton per sq. ft.
Third ring . . .	$2{\cdot}16 \times \pi \times \frac{10^2 - 5^2}{4} = 127$ tons	$\frac{7{\cdot}5}{25} = 0{\cdot}3$, $K_3 = 0{\cdot}37$	$\frac{0{\cdot}37 \times 127}{25 \times 25} = 0{\cdot}08$ ton per sq. ft.
Central area . .	$2{\cdot}00 \times \pi \times \frac{5 \times 5}{4} = 39$ tons	Nil, $K_4 = 0{\cdot}48$	$\frac{0{\cdot}48 \times 39}{25 \times 25} = 0{\cdot}03$ ton per sq. ft.
			Total pressure = 0·4 ton per sq. ft.

TABLE 9. (*Problem 74b.*)

Area.	Load on ring.	$\frac{d}{D}$.	K.	Stress at depth $= \frac{Kp}{D^2}$.
Outer ring . . .	$4 \times \frac{20^2 - 15^2}{4} = 550$ tons	0·7	0·18	$\frac{0{\cdot}18 \times 550}{25 \times 25} = 0{\cdot}158$ ton per sq. ft.
Second ring . . .	$4 \times \frac{15^2 - 10^2}{4} = 393$ tons	0·5	0·27	$\frac{0{\cdot}27 \times 393}{25 \times 25} = 0{\cdot}17$ ton per sq. ft.
Third ring . . .	$4 \times \frac{10^2 \times 5^2}{4} = 236$ tons	0·3	0·37	$\frac{0{\cdot}37 \times 236}{25 \times 25} = 0{\cdot}14$ ton per sq. ft.
Central area . . .	$4 \times 2{\cdot}5^2 = 78{\cdot}5$ tons	Nil	0·48	$\frac{0{\cdot}46 \times 78{\cdot}5}{25 \times 25} = 0{\cdot}04$ ton per sq. ft.
				Total pressure = 0·51 ton per sq. ft.

(*a*) *Rigid base.* Divide the circular base of the tower into four rings by means of concentric circles.

From equation (39) $q = \frac{p}{2\sqrt{1 - c^2}}$

Hence at base $q_1 = \frac{4}{2\sqrt{1 - 0{\cdot}875^2}} = 4{\cdot}55$ tons per sq. ft.

$$q_2 = \frac{4}{2\sqrt{1 - 0{\cdot}625^2}} = 2{\cdot}66 \quad ,, \quad ,,$$

$$q_3 = \frac{4}{2\sqrt{1 - 0{\cdot}375^2}} = 2{\cdot}16 \quad ,, \quad ,,$$

$$q_4 = 0{\cdot}5p \qquad = 2 \quad ,, \quad ,,$$

Table 8 can then be drawn up.

(*b*) *Flexible base.* Divide the area of the base as above into four areas, and from equations (37) or (38) Table No. 9 may be drawn up.

Maximum Shear Stress

The consideration of maximum shear stress is important in stability problems where there is a possibility of shear occurring in the soil, as may be associated with retaining walls, quay walls, embankments, etc. Excessive loads on a foundation will cause the soil beneath it to become plastic and cause shear failure. The foundation of a structure must be designed to distribute the load in such a way that the intensity of stress in the soil does not cause excessive settlement.

The determination of the maximum shear stress in the soil below the foundation of a structure is usually calculated as a plane problem either analytically or graphically by Mohr's circle. The results of these calculations are shown in the diagrams in Fig. 62.

The maximum shearing stress at any point P at a depth D caused by a single strip footing of uniform load p lb. per sq. ft. is :—

$$\text{Maximum} \quad S = \frac{p}{\pi}\sin 2a = \frac{p \times 2\frac{D}{b}}{\pi \times \left(1 + \left(\frac{D}{b}\right)^2\right)} \quad . \quad . \quad (40)$$

where b denotes the half-breadth of the foundation. The maximum value of the shear stress under such a footing is $\frac{p}{\pi}$ lb. per sq. ft. which occurs at points lying on a semi-circle of diameter equal to the footing width 2b. Directly under the centre of the

foundation the maximum shear occurs at a depth b, equal to half the width of the foundation.

The maximum shear stress under a triangular loaded strip is 0·256p lb. per sq. ft., where p is the unit load at the centre of

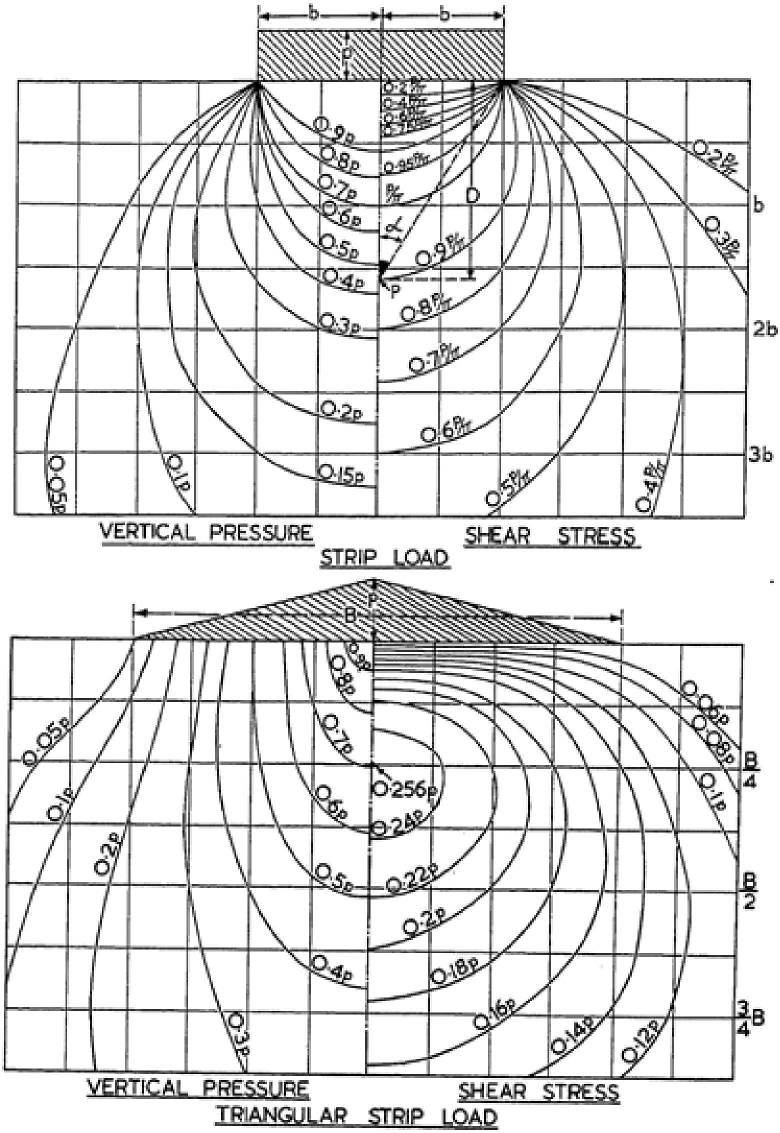


Fig. 62.—Distribution of Shear Stress and Vertical Pressure.

the loaded portion, and this occurs directly under the centre of the triangular load at a depth equal to a quarter of the width B of the strip.

Problem 75. A masonry bridge pier is 30 ft. high and 20 ft. wide. Calculate the shear stress and normal pressure in the clay soil at a depth of 15 ft. under the centre line of the pier, if the weight of the masonry is 140 lb. per cu. ft. (Assume the load to be a strip load and neglect live loading.)

Shear stress (from equation 40)

$$s = \frac{4200}{\pi} \times \frac{2 \times \frac{15}{10}}{1 + \left(\frac{15}{10}\right)^2}$$

$$= \frac{4200 \times 120}{\pi \times 130} = 1{,}234 \text{ lb. per sq. ft.}$$

(Check by Fig. 61) $s = 0{\cdot}92 \times \frac{p}{\pi} = 1{,}235$ lb. per sq. ft.

Normal pressure.

(From Fig. 62.) At depth 15 ft. or 1·5b, pressure = 0·48p.
$q = 0{\cdot}48 \times 4200 = 2{,}016$ lb. per sq. ft.

Problem 76. A triangular embankment is to be constructed with material weighing 110 lb. per cu. ft. The base is 108 ft. wide and the height 36 ft. The embankment is to be founded on an existing deep deposit of clay soil of shear strength 800 lb. per sq. ft. Calculate the maximum shear occurring beneath the embankment foundation, ascertain the depth and whether the shear strength of the clay is exceeded.

From Fig. 62, the maximum shear stress is 0·256p occurring at the depth 0·25B.

Max. $S = 0{\cdot}256 \times 110 \times 36$ lb. per sq. ft. at a depth 27 ft.
$= 1{,}013$ lb./sq. ft. at depth 27 ft.

It will be apparent that the shear strength of the existing clay will be exceeded. It may be expected, therefore, that plastic flow will occur in the clay under the new embankment and considerable settlement will take place with a heave of the existing soil taking place beyond the toe of the embankment.

CHAPTER X

FOUNDATIONS—BEARING CAPACITY (*Continued*)

Ultimate Bearing Capacity—Cohesive Soils

The previous chapter describes the distribution of vertical pressure and shear stress in a soil underlying the foundation of a structure.

The present chapter refers to the calculation of the ultimate bearing capacity of soils. Investigations by Professor L. Prandtl and Dr. H. Hencky on surface loading of cohesive soils indicate that the failure of a long strip footing occurs by a wedge of material directly under the foundation being forced downwards, which causes the soil on either side to flow outwards and a heaving

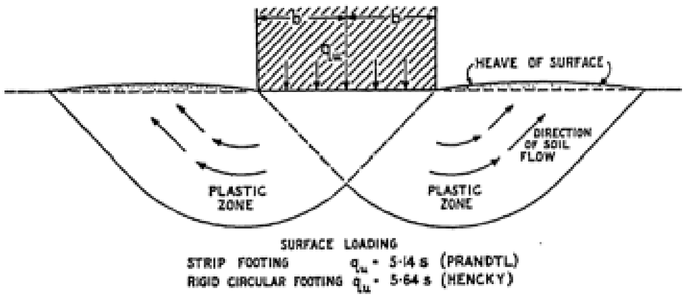


FIG. 63.—Bearing Capacity of Cohesive Soils.

of the surface to take place on each side of the footing, with a consequent settlement of the structure, as indicated in Fig. 63. If a graph is plotted for the amount of settlement against the intensity of loading, it will be found that after a certain load has been applied the settlement becomes excessive, and this intensity of loading is known as the ultimate bearing capacity.

Considering surface loading on a homogeneous clay, Prandtl found that the ultimate bearing capacity for a strip footing is

$$q_u = (2 + \pi)s = 5{\cdot}14s \quad . \quad . \quad . \quad . \quad (41)$$

For a rigid circular footing Hencky states that

$$q_u = 5{\cdot}64s \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

In each case s denotes the shear strength of the clay.

For footings below the surface Mr. A. W. Skempton has developed the following formula for homogeneous clays, which takes into account the adhesion of the soil along the sides of the footing :—

The ultimate bearing capacity,

$$q_u = \text{the gross pressure on the clay } q, \text{ less } \gamma D.$$

Hence,
$$q = 5{\cdot}64\bar{s} + \frac{Fs'}{A} + \gamma D \quad . \quad . \quad . \quad . \quad (43)$$

where D denotes the depth of the foundation below the surface,
- γ denotes the unit weight of the clay.
- F denotes the area of the side of the footing in contact with the clay,
- A denotes the area of the base of the footing,
- s denotes the shear strength of the clay, and
- s′ denotes the skin friction or adhesion between the soil and the footing, and may be taken as being $0{\cdot}75\bar{s}$.

This equation gives the ultimate bearing capacity causing complete failure of the soil, and it is necessary to introduce a factor of safety to keep the settlement within small limits. It is recommended that a factor of safety of 2 be adopted generally for ordinary buildings, but this should be increased to 3 for structures subjected to vibration and for sensitive buildings such as rigid frame, monolithic frame or flexible frame with light stone facings.

Problem 77. A circular bridge pier 20 ft. in diameter is founded at a depth of 30 ft. in a clay soil of shear strength 400 lb. per sq. ft. and unit weight of 120 lb. per cu. ft. Ascertain the ultimate bearing capacity and the safe total load if a factor of safety of 2 is adopted. Compare the safe total load on the pier if founded at ground level.

From equation (43) the ultimate bearing capacity q_u

$$= (q - \gamma D) = 5{\cdot}64\bar{s} + \frac{Fs'}{A} = (q - 120 \times 30)$$
$$= (5{\cdot}64 \times 400) + \left(\frac{30 \times 3{\cdot}14 \times 20}{3{\cdot}14 \times 10 \times 10} \times \tfrac{3}{4} \times 400\right)$$
$$= 2256 + 1800$$
$$= 4{,}056 \text{ lb. per sq. ft.} = 1{\cdot}81 \text{ tons per sq. ft.}$$

The gross loading on the clay

$$= q = q_u + \gamma D$$
$$= 4056 + 3600$$
$$= 7{,}656 \text{ lb./sq. ft.} = 3{\cdot}42 \text{ tons/sq. ft.}$$

Allowing factor of safety of 2, safe loading = 1·71 tons per sq. ft.

Hence, the safe total = area of base × 1·71
= 3·14 × 10 × 10 × 1·71
= 537 tons

For surface loading, the ultimate bearing capacity q_u

= 5·64s̄ = 1·01 tons per sq. ft.
the safe bearing capacity = 1·01 × 0·5 = 0·51 ton per sq. ft.
and the safe total load = 3·14 × 10 × 10 × 0·51 = 159 tons.

Problem 78. The footing of a reinforced concrete column is 8 ft. square and 2 ft. 6 ins. thick, and is founded at a depth of 6 ft. in a homogeneous clay soil of shear strength 700 lb. per sq. ft. and unit weight of 110 lb. per cu. ft. Calculate the safe total load on this footing, adopting a factor of safety of 2.

From equation (43) the ultimate bearing capacity q_u

$$= (q - 110 \times 6) = (5 \cdot 64 \times 700) + \left(\frac{2 \cdot 5 \times 32 \times 3 \times 700}{8 \times 8 \times 4}\right)$$
$$= 3948 + 656 = 4{,}604 \text{ lb. per sq. ft.}$$

The gross loading on the clay

$$= q = 4604 + 660 = 5{,}264 \text{ lb. per sq. ft.}$$

Allowing factor of safety of 2, safe loading = 2,632 lb. per sq. ft.

Hence the safe total load = area of base × 2632
= 2632 × 8 × 8 lb.
= 75·2 tons

Circular-Arc Method for Foundations

Where the shear strength of the soil below a footing varies with the depth, the graphical circular-arc method of Fellenius may be used. This method assumes the clay will fail by shearing along a cylindrical arc, as indicated in Fig. 65. The surface of failure is taken to pass through one lower corner of the loaded area and with the centre of the arc near the ground surface at some distance from the opposite side of the loaded area. Moments are taken about this centre, and for stability.

$$W \times z = L \times \bar{s} \times R \quad . \quad . \quad . \quad . \quad (44)$$

where W denotes the weight of the foundation load of unit thickness,
- L denotes the length of the arc AB,
- R denotes the radius of the cylindrical surface of shear,
- $\bar{s}$ denotes the average shear strength of the soil, and
- z denotes the distance of the centre of the foundation from centre O.

From this equation we can derive the factor of safety as :

$$G = \frac{L \times s \times R}{W \times z}$$

and generally a value between 1·5 and 2 is adopted.

The critical circle may be found by trial and error, and in a non-homogeneous clay any point is chosen as a centre, and the average shear strength along the arc is calculated by taking the clay as being uniform over small strips. This is repeated for different centres until the minimum value of the footing load is obtained. Mr. Guthlac Wilson has determined the position of the

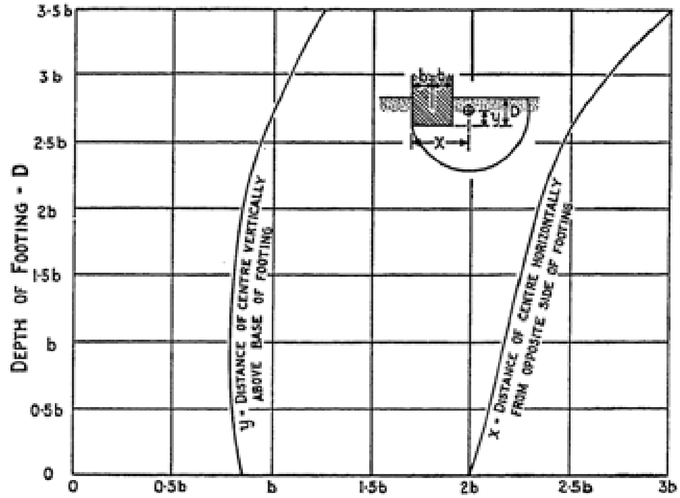


FIG. 64.—G. Wilson's Analysis for Co-ordinates of Critical Circle.

centre O analytically for a homogeneous clay, and this method is useful for obtaining the position for the first trial centre. The graph in Fig. 64 will enable this initial trial centre to be determined, but a variation in the position of this centre will occur dependent on the variation in the shear strength of the underlying clay. The Wilson method assumes that the soil has shrunk away from one side of the footing AC and there is no adhesion between the soil and this side of the footing. The bearing capacity for surface loading with the circular-arc method in homogeneous clay is 5·52s, which agrees satisfactorily with Hencky's result of 5·64s in equation (42).

Problem 79. The total load on a stanchion is 100 tons and the footing is spread to 8 ft. by 9 ft. The concrete base is 2 ft. 6 ins.

thick, and founded at a depth of 5 ft. 6 ins. in clay of varying shear strength, as shown in Fig. 66. Adopting the circular arc method, and assuming Wilson's centre as the centre of the

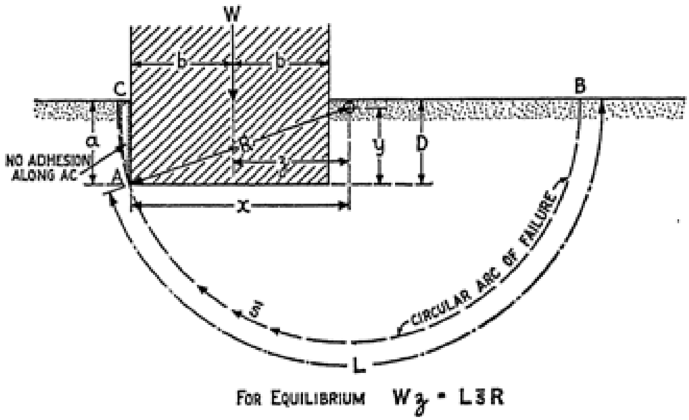


Fig. 65.—Bearing Capacity of Clay Soil—Circular-arc Analysis.

critical circle, calculate the factor of safety against shear failure of the soil under the foundation. Compare with Hencky's formula and Skempton's method.

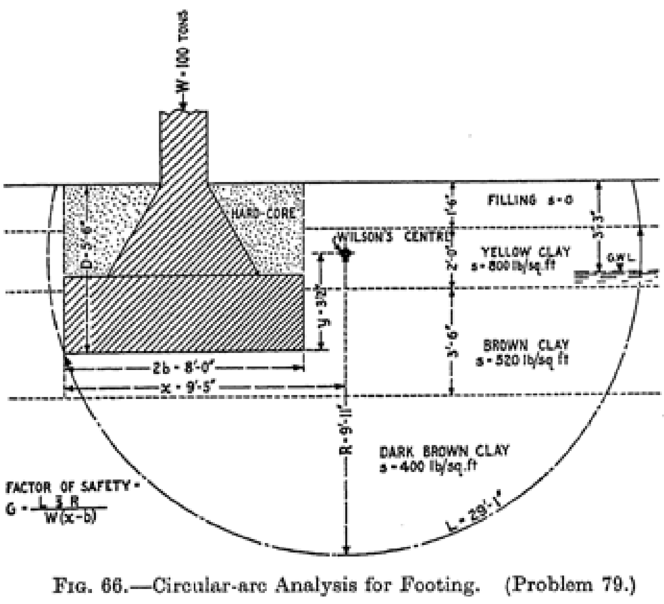


Fig. 66.—Circular-arc Analysis for Footing. (Problem 79.)

With reference to Fig. 66, from the graph in Fig. 64, the co-ordinates of the critical circle will be :—

$$D = \frac{5{\cdot}5b}{4} = 1{\cdot}375b$$

$$\text{Hence, } x = 9 \text{ ft. } 5 \text{ ins.}$$
$$\text{and } y = 3 \text{ ft. } 2 \text{ ins.}$$

This determines the centre O, and with radius OA (see Fig. 65) the circular arc is drawn as shown in Fig. 66.

From equation (44) the factor of safety, $G = \dfrac{L \times \bar{s} \times R}{W \times z}$

$$= \frac{(L_1 s_1 + L_2 s_2 + \dots\dots\dots)R}{W \times z}$$

where L_1, L_2, etc., are the lengths of the arc passing through strata of shear strengths s_1, s_2, etc.

The following table is then drawn up :—

Soil.	Length of arc, L.	Shear strength, s.	L × s, ft. lb.	Other data.
	ft. in.			
Brown Clay .	1 7	520 lb./sq. ft.	823	$W = \frac{100}{9} = 11{\cdot}11$ tons.
Dark Brown Clay .	21 9	400 „	8,700	$z = (x - b)$ $= 5$ ft. 5 ins.
Brown Clay .	3 9	520 „	1,950	$R = 9$ ft. 11 ins.
Yellow Clay .	2 0	800 „	1,600	
Filling .	1 6	Nil	—	
	30 7	Average value 427 lb./sq. ft.	13,073	

$$\text{Hence, factor of safety, } G = \frac{13{,}073 \times 9 \text{ ft. } 11 \text{ ins.}}{11{\cdot}11 \times 2240 \times 5 \text{ ft. } 5 \text{ ins.}} = 0{\cdot}97.$$

This result indicates that the foundation is unstable, and adopting a factor of safety of 2, the safe load for this stanchion would be 50 tons.

The following is a comparison with the results obtained from the formulæ of Hencky and Skempton :—

(*a*) Hencky's formula for surface loading (42)

$$\text{Factor of safety} = \frac{5{\cdot}64s \times 72}{2240 \times 100} = 0{\cdot}8$$

(*b*) Skempton's formula (43)

$$\text{Factor of safety} = \frac{q \times 72}{2240 \times 100} = \frac{3419 \times 72}{2240 \times 100} = 1{\cdot}0$$

$$q = (5{\cdot}64 \times 427) + (100 \times 5{\cdot}5) + \frac{(34 \times 2{\cdot}5 \times 3 \times 520)}{8 \times 9 \times 4}$$

$$= 3{,}419 \text{ lb. per sq. ft.}$$

Raft Foundations—Over-stressed Zone

Although a factor of safety between 1·5 and 2 may have been adopted, it is possible that the maximum shearing stress under the foundation may exceed the shear strength of the soil at certain points, causing a tendency for a plastic zone to develop. It is desirable, therefore, that the footing load should be limited so that at no point does the shear stress exceed the shear strength of the soil. The variation of the shear strength with depth is

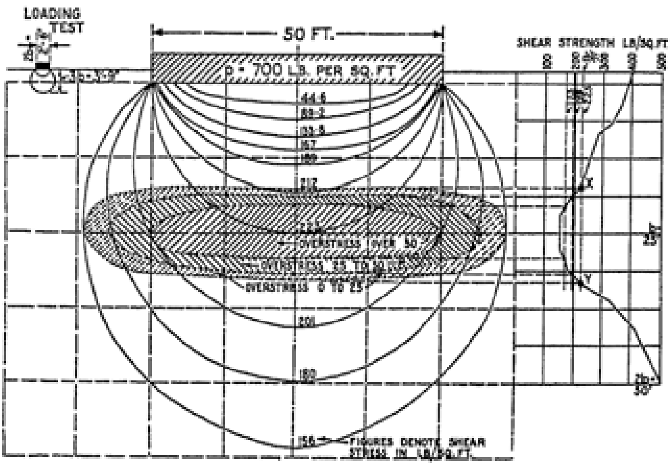


FIG. 67.—Raft Foundation. Over-stressed Zone. (Problem 80.)

plotted, and when this is compared with the theoretical shear stresses due to the structural load, it may be found that the soil is over-stressed in shear within a zone which is known as an over-stressed zone, as indicated in Fig. 67. Such a zone is supported by the stronger stratum of clay above it, and complete failure is prevented, but a certain amount of plastic yield takes place, resulting in excessive settlement of the structure.

This method of design is essential in connection with raft foundations. The size of the over-stressed zone will enable some idea to be gained of the settlements to be expected, but if the

soil is nowhere over-stressed it may be assumed that only consolidation settlement will occur.

Problem 80. A structure 50 ft. wide has a uniformly distributed load of 700 lb. per sq. ft. The underlying strata varies in shear strength from 400 lb. per sq. ft. to 150 lb. per sq. ft., as indicated in the diagram, Fig. 67. Ascertain the over-stressed zones below the structure.

Below the raft foundation isolines joining points of equal shear stress are plotted as shown in Fig. 67, and to one side of this diagram a chart is drawn indicating the variation of shear strength with depth.

On the latter chart a vertical line is drawn representing $\frac{p}{\pi}=\frac{700}{3\cdot 14}=223$ lb. per sq. ft., which cuts the graph of shear strength at the points X and Y. The projection of X and Y to points on the perpendicular bisector of the base indicates the depths between which over-stressing will occur, and by interpolation the zone of over-stress can be traced.

As shown in Fig. 67, zones of over-stress between zero and 25 lb. per sq. ft., 25 to 50 lb. per sq. ft. and over 50 lb. per sq. ft. may be drawn. It will be evident that in this problem a large area of the underlying strata will be considerably over-stressed to a maximum of 48 per cent., and extensive settlement will occur due to the development of a plastic zone beneath the footing.

Note on Loading Tests on Clay

The expense of loading tests on clay cannot be justified now that borings and compression tests can be easily and economically carried out. If the clay is homogeneous and of considerable depth, the results obtained from a loading test can be useful, but in a large number of cases they may be misleading. From Fig. 67, it will be realised that a structure of width 2b will influence the soil to a depth of at least 3b, whereas a loading test with its smaller area would produce only data relating to soil strata near the surface.

This is particularly an important feature at a site where a layer of soft clay may exist under a stratum of medium strength clay. In such a circumstance a loading test does not cause the soft clay to be stressed to any marked degree whereas, a structure of width equal to twice the depth of this soft layer would cause a shear stress of $\frac{p}{\pi}$ to be set up, which might cause plastic flow to occur, of which possibility the loading test would provide no information.

On the other hand, by means of site exploration with post-hole auger or boring tackle and taking samples for compression or shear-box tests, a detailed analysis can be made of the most complex of site conditions, enabling all the variations of strata to be studied in a way which would be impossible with loading tests, and in a more economical manner.

Ultimate Bearing Capacity—Cohesionless Soils

The bearing capacity of sands differs from that of clays, inasmuch as the limiting conditions depend on settlement, and in most instances serious settlement occurs before the ultimate bearing capacity is reached.

Further experiments are needed to establish a reliable method for calculating the bearing capacity of sands, but at the present time, until additional information is forthcoming, the results obtained from Ritter's equation, as given below, furnishes the most satisfactory results, although the values obtained are conservative.

Ultimate bearing capacity (for cohesionless soils)

$$= q_u = \left[\gamma D + \frac{\gamma B}{4}\tan\left(45^\circ + \frac{\phi}{2}\right)\right]\left[\tan^4\left(45^\circ + \frac{\phi}{2}\right) - 1\right] + \gamma D \quad . \quad . \quad (45)$$

where D denotes the depth of the foundation,
B denotes the breadth of the foundation,
γ denotes the unit weight of the soil, and
ϕ denotes the angle of internal friction of the soil.

Problem 81. A foundation in a dense sand is 12 ft. wide and 5 ft. deep. The soil weighs 110 lb. per cu. ft. and has an angle of internal friction of 38°. What is the safe bearing capacity, adopting a factor of safety of 2? Compare with the safe loading capacity for surface loading?

Ultimate bearing capacity (from equation 45)

$$q_u = \left[(110 \times 5) + \left\{\frac{110 \times 12}{4}\tan\left(45^\circ + \frac{38^\circ}{2}\right)\right\}\right]\left[\tan^4\left(45^\circ + \frac{38^\circ}{2}\right) - 1\right] + (110 \times 5)$$

$$= \left[550 + \left\{\frac{1{,}320}{4}\tan 64^\circ\right\}\right]\left[\tan^4 64^\circ - 1\right] + 550$$

$$= [550 + (330 \times 2{\cdot}05)](2{\cdot}05^4 - 1) + 550$$

$$= (1{,}226.5 \times 16{\cdot}64) + 550$$

$$= 20{,}959 \text{ lb. per sq. ft.} = 9{\cdot}4 \text{ tons/sq. ft.}$$

$\therefore$ Safe bearing capacity, $q = 4{\cdot}7$ tons per sq. ft.

For surface loading equation (45) becomes

$$q_u = \left[\frac{\gamma B}{4}\tan\left(45^\circ + \frac{\phi}{2}\right)\right]\left[\tan^4\left(45^\circ + \frac{\phi}{2}\right) - 1\right]$$

$$= \left[\frac{110 \times 12}{4}\tan 64^\circ\right]\left[\tan^4 64^\circ - 1\right]$$

$$= 676{\cdot}5\ (17{\cdot}64 - 1)$$

$$= 11{,}257 \text{ lb./sq. ft.} = 5 \text{ tons/sq. ft.}$$

$\therefore$ Safe bearing capacity, q = 2·5 tons/sq. ft.

Problem 82. A foundation of width 10 ft. and depth of 4 ft. is to be founded in a sand with an angle of internal friction of 32° and unit weight of 100 lb. per cu. ft. Calculate the safe bearing pressure adopting a factor of safety of 3 as the structure will be subject to vibration.

From equation (45)

$$q_u = \left[(100 \times 4) + \left\{\frac{100 \times 10}{4}\tan\left(45^\circ + \frac{32^\circ}{2}\right)\right\}\right]\left[\tan^4\left(45^\circ + \frac{32^\circ}{2}\right) - 1\right] + (100 \times 4)$$

$$= [400 + 250 \tan 61^\circ][\tan^4 61^\circ - 1] + 400$$

$$= (400 + 450)\ (10{\cdot}5 - 1) + 400$$

$$= 8475 \text{ lb./sq. ft.} = 3{\cdot}8 \text{ tons/sq. ft.}$$

$\therefore$ Safe bearing capacity, $q = \frac{3{\cdot}8}{3} = 1{\cdot}3$ tons/sq. ft.

CHAPTER XI

FOUNDATIONS—SETTLEMENTS DUE TO CONSOLIDATION

It may be as well to review the data relating to the design of foundations as contained in Chapters IX and X and continued in the present chapter.

Design depends on two main factors :—

(1) *Shear strength.* The shear strength of the soil must not be exceeded. The determination of the shear stress due to any loading can be calculated from Equation (40), and the distribution of stress obtained from Fig. 62. (The method is illustrated in Problems 75 and 76.)

The ultimate bearing capacity of clays can be calculated in the manner referred to in Chapter X from Equations (41), (42) and (43), and examples are given in problems 77 and 78. The circular-arc method may be adopted as an alternative as shown in Problem 79.

Investigation into over-stressed zones for raft foundation design is made in Problem 80. The bearing capacity of sands is calculated from Equation (45) as indicated in Problems 81 and 82.

(2) *Consolidation.* Excessive settlement or large differential settlements due to consolidation of the soil must be avoided. In this case the calculation of vertical pressure at any depth with strip footings is made by means of Equations (37) and (38), and the graph in Fig. 61, as illustrated in Problems 68 and 72. For circular loaded areas, Equation (39) is used for rigid bases, and Equations (37) and (38) for flexible bases.

The present chapter will investigate further the settlement due to consolidation, and equations will be given enabling such settlements to be calculated. Such data will make it possible for precautions to be taken against excessive variation of settlement over the area of the foundation of a structure.

Generally, settlements due to shear failure take place rapidly as soon as the ultimate bearing capacity is exceeded, and usually while the structure is being built. Settlements due to consolidation, however, continue for years after the completion of the structure, and may be appreciable to the extent of causing

failure even although the pressure is much below the allowable value.

Settlement in Clays—Consolidation Test

The compressibility of most clays under applied loads is appreciable, and since a reduction in volume can take place only

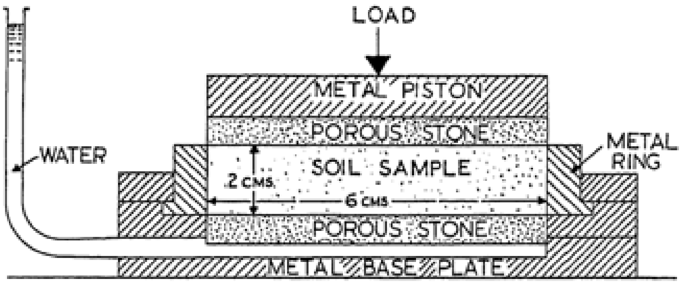


Fig. 68.—Consolidation Test in Œdometer.

by giving up some of the pore water, the rate at which compression occurs is very slow owing to the low permeability. This slow

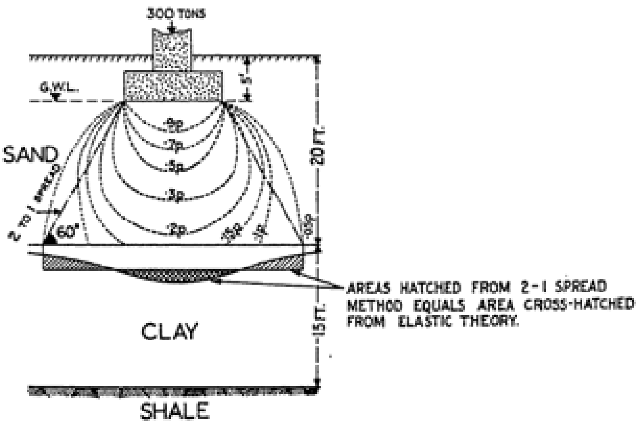


Fig. 69.—Vertical Pressure. Load Spread. (Problem 83.)

volume reduction under pressure is referred to as consolidation, and the main features may be illustrated by means of consolidation test results.

Clays consist of a framework of small mineral particles, generally arranged in a loose state of packing, the intervening spaces being filled with water. When a load is applied to the surface of a clay layer the particles in the stressed zone will move into a closer state of packing and some water will be extruded. Eventually equilibrium will be reached under the applied load, the voids ratio of the clay having been reduced and the shear strength increased.

Consolidation tests are carried out in an Œdometer, as shown in diagram Fig. 68. The specimen from an undisturbed sample of clay measures 6 cm. in diameter and 2 cm. thick, and is confined laterally in a metal ring. The specimen is compressed between two porous plates, which are kept saturated with water during a

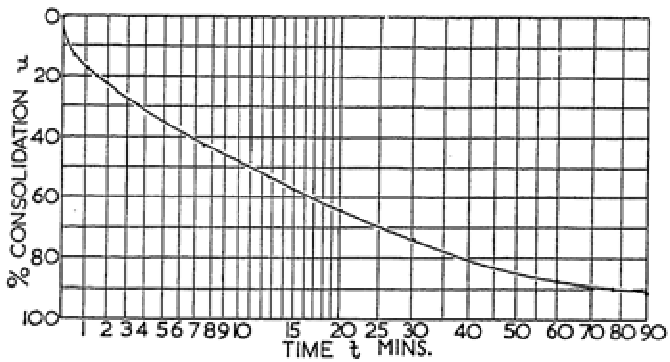


FIG. 70.—Percentage Consolidation/Time Curve. (Problem 83.)

test. A load is applied to the upper stone, and the clay consolidates, the excess pore water escaping through the porous stones. After each increment of load is applied, it is allowed to remain on the sample for a sufficient time to allow equilibrium to be established, and a "consolidation curve" showing the deformation with time is obtained for each increment as shown in Fig. 70.

The compression of the clay is measured by means of a micrometer dial. Movement, which is at first rapid, will usually have ceased after twenty-four hours, the clay then being in equilibrium at the new pressure.

When the last load increment has been applied and the resulting consolidation measured, the load increments are removed and the expansion of the soil measured. When a soil sample is taken for test there is a relief of pressure, and swelling occurs after extraction, so that the voids ratio of the sample is greater than the voids ratio of the soil layer. It is necessary, therefore, to estimate the value

of the voids ratio from the initial consolidation due to the overburden.

Theory of Consolidation

From the equilibrium values in the consolidation test a curve can be drawn showing the relation between the voids ratio, e, and the applied pressure, p, which is known as the p–e curve or laboratory compression curve, as illustrated in Fig. 71. From this curve we obtain the following equation :—

$$\left.\begin{array}{l}\text{Settlement in a clay layer}\\ \quad\text{of thickness, H.}\end{array}\right\} = \Delta H = \frac{e_1 - e_2}{1 + e_1} H \quad . \quad . \quad . \quad (46)$$

The time required to reach a given degree of consolidation is directly proportional to the square of the thickness of the layer, H_0; inversely proportional to the coefficient of permeability, K; and directly proportional to the slope of the p–e curve, a.

The slope of the p–e curve, a, is known as the coefficient of compressibility and

$$a = -\frac{de}{dp} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

which may be ascertained from the p–e curve. Usually the average value of a is found by assuming a straight line joining two points corresponding to successive load increments.

The following equations can then be derived :—

The coefficient of permeability, $K = K_0 (1 + e)$ . . . (48)
where K_0 is a constant value and e is the voids ratio at any time.

The coefficient of consolidation, $C = \frac{K_0}{a}$ . . . . . (49)

The percentage of consolidation, $u = \int \left(\frac{ct}{H_0^2}\right)$

where H_0 is the thickness of the clay if drainage occurs on one side. (If drainage occurs on both sides of the clay layer, then the value of $\frac{H_0}{2}$ should be used.)

Thus, as expressed by the time/settlement curve, the "time factor,"

$$T = \frac{ct}{H_0^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

The percentage consolidation, u, plotted against T, gives the

theoretical rate of consolidation, and the following table (10) gives values of u for increments of 5% and T :—

TABLE 10.

u%.	T.	u%.	T.	u%.	T.	u%.	T.
0	0	30	0·072	55	0·238	80	0·565
10	0·008	35	0·097	60	0·287	85	0·684
15	0·017	40	0·126	65	0·342	90	0·848
20	0·031	45	0·159	70	0·405	95	1·127
25	0·049	50	0·195	75	0·475	100	—

Variation of Pressure in Thick Clay Strata.

The foregoing formulæ apply to clay layers which are not of exceptional thickness, and it is assumed that the load acting throughout the soil layer is constant. If the thickness of the clay stratum is large, the load may increase or decrease from the upper surface to the lower surface. In such cases a factor N is used instead of the " time factor," T, and $N = \frac{\pi^2 T}{4}$ and the following table (11) gives values for u and N for the three cases which might occur, viz.,

Case A, where uniform pressure exists.

Case B, where triangular pressure varying uniformly from zero at the top surface to a maximum at the lower surface.

Case C, where pressure varies uniformly from a maximum at the top to zero at the bottom of the stratum.

Where trapezoidal pressure exists, Cases B, or C, enable the appropriate value for N to be ascertained and this value is added to that for Case A.

TABLE 11.

u%.	N.			u%.	N.		
	A.	B.	C.		A.	B.	C.
5	0·005	0·06	0·002	50	0·49	0·73	0·24
10	0·02	0·12	0·005	55	0·59	0·84	0·32
15	0·04	0·18	0·01	60	0·71	0·95	0·42
20	0·08	0·25	0·02	65	0·84	1·10	0·54
25	0·12	0·31	0·03	70	1·00	1·24	0·69
30	0·17	0·39	0·06	75	1·18	1·42	0·88
35	0·24	0·47	0·07	80	1·40	1·64	1·08
40	0·31	0·55	0·13	85	1·69	1·93	1·36
45	0·39	0·63	0·18	90	2·09	2·35	1·77
				95	2·80	3·17	2·54

The time in years, t, corresponding to any percentage consolidation may be found from the following equation :—

$$t = \frac{NH^2}{1400 \times C} = \frac{TH^2}{5672 \times C} \quad . \quad . \quad . \quad (51)$$

where C is in sq. cms. per minute and H is the reduced thickness in feet.

It will now be realised that if the thickness and boundary drainage conditions of a clay stratum are known, then the laboratory consolidation curve can be converted into a curve showing the probable progress of settlement with time of any structure loading the clay layer. The application will be dealt with in detail in the problems to follow.

Design with Respect to Consolidation

Calculation is first made of the ultimate settlement at the centre of the loaded area, and should this be found to be excessive, then the time required for 90 per cent. of the settlement should be ascertained. If this time considerably exceeds the probable life of the building, then the probable settlement during the life of the structure should be calculated.

When the settlement during the life of the structure is considerable, it is necessary to reduce the bearing pressure by adopting a greater spread of the foundations or providing a raft, and the settlement is re-calculated for the new conditions. If the settlements are still excessive, then another type of foundation will need to be designed either by adopting piles or a floating foundation.

If it is found that the maximum settlement is not too great, then the next step is to investigate relative settlements of various parts of the foundation, and, where excessive variations occur, it may be possible to equalise the loading to make the settlements of the same order. Alternatively, a rigid raft may be designed, as with such a foundation the bearing pressures at the corners and edges are higher than at the centre, and in order to attain sufficient rigidity it may be necessary to construct a stiff cellular type of raft.

Problem 83. It is proposed to construct a building 80 ft. square, supported by five columns in each direction, spaced at 20 ft. centres. Each column will have a 10-ft.-square footing founded at 5 ft. below ground level, and it is estimated that a load of 300 tons will be carried by each column. The foundation consists of a sand overlying a soft clay, which occurs at a depth of 20 ft. below the footing base. The soft clay layer is 15 ft. thick, and a close shale underlies the clay. It may be assumed that moisture will not flow into the shale. Ground-water level occurs

at 5 ft. below ground level. Adopting the p–e curve shown in Fig. 71, for a sample 1 in. thick and the time/settlement curve shown in Fig. 70, calculate the ultimate settlement and the time in which 50 per cent. of the settlement will occur. (Weight of dry clay soil is 112 lb. per cu. ft.)

For a base 10 ft. square carrying a load of 300 tons, the average pressure on the soil at the base is 3 tons/sq. ft., but consideration must be given to an unequal distribution of pressure as referred to in Chapter IX.

The distribution of vertical pressure may be calculated from

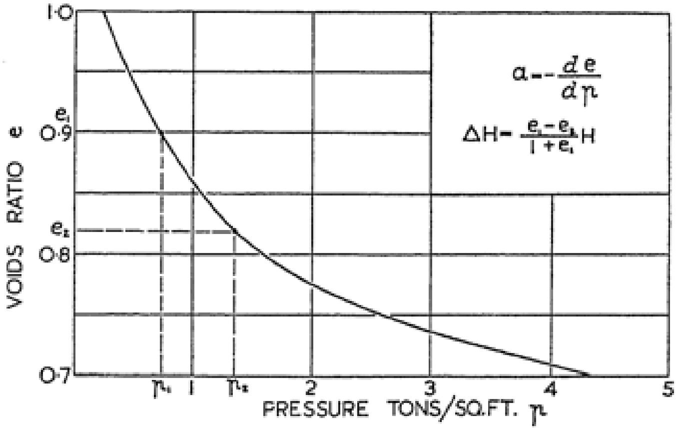


FIG. 71.—p–e Curve. Laboratory-Compression Curve. (Problem 83.)

Equation (37) Chapter IX, or ascertained from Fig. 62, but generally for rigid footings a load spread of 2 to 1 may be adopted. The spread of 2 to 1 is derived from the distribution of vertical pressure as given by the elastic theory. The rectangle of pressure obtained by the "spread" method is for all practical purposes equal to the area bounded by the curved line derived from the theory of elasticity as indicated in Fig. 69.

Hence the pressure on the clay at a depth of 20 ft. below the footing may be considered to be spread over an area 30 ft. square, thus,

$$q = \frac{300}{30 \times 30} = 0{\cdot}33 \text{ ton per sq. ft.}$$

(Check : From Fig. 62, Chapter IX, the vertical pressure for a strip load at a depth of 4b (= 20 ft.) may be estimated to be $0{\cdot}1p = 0{\cdot}1 \times 3 = 0{\cdot}3$ ton/sq. ft.)

Where the footings are spaced at intervals, the spread of pressure from each footing overlaps. The total area of spread

for the whole building = (15 ft. + 80 ft. + 15 ft.) = 110 sq. ft., and the average vertical pressure on the clay,

$$q = \frac{25 \times 300}{110 \times 110} = 0{\cdot}62 \text{ ton per sq. ft.}$$

(It will be noted that this pressure is practically double that for one footing base.)

The original pressure on the clay before loading with the new structure is that due to the sand, which is

$$\frac{[20 \times (112 - 62)] + (5 \times 112)}{2240} = 0{\cdot}7 \text{ ton/sq. ft.}$$

allowing for buoyancy.

Hence, the total pressure due to the overlying strata and the structure is

$$0{\cdot}62 + 0{\cdot}7 - \left(\frac{5 \times 112}{2240}\right) = 1{\cdot}07 \text{ ton/sq. ft.}$$

At the centre of the clay stratum the pressure due to the overburden

$$\frac{[(20 + 7{\cdot}5)(112 - 62)] + (5 \times 112)}{2240} = 0{\cdot}864 \text{ ton/sq. ft.}$$

The total pressure at the centre of the clay stratum $= 0{\cdot}62 + 0{\cdot}864 - 0{\cdot}25 = 1{\cdot}234$ tons/sq. ft.

From the p–e curve (Fig. 71) if the original pressure $p_1 = 0{\cdot}864$ ton/sq. ft., then $e_1 = 0{\cdot}90$, and if the total pressure $p_2 = 1{\cdot}234$ tons/sq. ft., then $e_2 = 0{\cdot}82$.

The ultimate settlement in a clay layer of 15 ft. thickness is

$$\Delta H = \frac{e_1 - e_2}{1 + e_1} H = \frac{0{\cdot}90 - 0{\cdot}82}{1 + 0{\cdot}90} 15 = 0{\cdot}63 \text{ ft.} = 7{\cdot}5 \text{ ins.}$$

This is the ultimate settlement which may extend over a period of fifty years, but the latter half of the settlement will take place at such a slow rate that most structures can adapt themselves to the movement. From laboratory experiments the time/settlement curve indicates that half the ultimate settlement occurs in ten minutes.

The degree of consolidation, $u = \int\left(\frac{ct}{H_0^2}\right)$, or the ratio of consolidation of a clay layer is roughly inversely proportional to H_0^2, where H_0 is the drainage path, or the longest distance a particle of water must travel to a free draining surface. In the laboratory test, H_0 is half the thickness of the sample which is 1 in. thick, but in the problem H_0 is equal to 15 ft., as the drainage

path from the impervious shale to the free draining surface into the sand stratum is the full thickness of the clay layer.

Hence, where $H_0 = \frac{1}{2}$ in. (by experiment), 50 per cent. consolidation occurs in ten minutes and, where $H_0 = 15$ ft. (present problem) 50 per cent. of consolidation occurs in

$$\frac{10 \times 15^2 \times 144}{0{\cdot}5 \times 0{\cdot}5 \times 60 \times 24} = 900 \text{ days} = 2{\cdot}5 \text{ years.}$$

Therefore, settlement amounting to 3·75 ins. will occur in two-and-a-half years after the construction of the building.

Problem 84. The following data was recorded from a consolidation test on a clay sample :—

Load, lb.	Dial reading, ins.	Load, lb.	Dial reading, ins.	Time, mins.
0	0	1600	0·0489	0
100	0·019		0·0576	0·25
200	0·0263		0·0586	1
400	0·0387		0·0595	4
800	0·0489		0·0603	15
1600	0·0638		0·0613	75
0	0·0392		0·0621	170
			0·0633	460
			0·0638	600

Area of sample = 12·57 sq. ins. Thickness of sample = 1·25 ins.
Specific gravity of sample = 2·67.
Final weight of wet sample = 507·3 grs.
Weight of dry sample = 412·5 grs.

Draw p–e and percentage consolidation/time curves and determine the coefficients of consolidation and compressibility for the clay sample.

Weight of water at end of test = 507·3 − 412·5 = 94·8 grs.

Volume of voids at end of test = 94·8 c.c.

$$\text{Volume of solid material} = \frac{412{\cdot}5}{2{\cdot}67} = 157{\cdot}5 \text{ c.c.}$$

Hence, height of soil sample,

$$H = \frac{157{\cdot}5}{12{\cdot}57 \times 2{\cdot}54^2} = 1{\cdot}948 \text{ cm.} = 0{\cdot}747 \text{ in.}$$

and height of voids at end of test

$$= \frac{94{\cdot}8}{12{\cdot}57 \times 2{\cdot}54^2} = 1{\cdot}17 \text{ cm.} = 0{\cdot}461 \text{ in.}$$

also height of voids at commencement of test, $M = 0{\cdot}461 + 0{\cdot}0392 = 0{\cdot}5002$ in.

The following table (12) can then be prepared.

TABLE 12.

Load increment.	Total Load.	Time interval.	Dial reading amount of consolidation, D.	Difference in dial reading over time interval.	Height of voids, M—Dial reading, D.	Percentage compression.	Voids Ratio.	Load, lb. per sq. in.
0	0	—	0	—	0·5002	—	0·67	0
1	100	—	0·019	—	0·4812	—	0·645	7·95
2	200	—	0·0263	—	0·4739	—	0·634	15·9
3	400	—	0·0387	—	0·4615	—	0·618	31·8
4	800	—	0·0489	—	0·4513	—	0·605	63·6
5	1600	0	0·0489	0	—	0	—	127·2
—	—	0·25	0·0576	0·0087	—	58·5	—	—
—	—	1	0·0586	0·0097	—	65·0	—	—
—	—	4	0·0595	0·0106	—	71·2	—	—
—	—	15	0·0603	0·0114	—	76·5	—	—
—	—	75	0·0613	0·0124	—	83·4	—	—
—	—	170	0·0621	0·0132	—	88·6	—	—
—	—	460	0·0638	0·0144	—	96·7	—	—
—	—	600	0·0638	0·0149	0·4304	100	0·584	—
0	0	—	0·0392	—	0·4610	—	0·617	0

From this data curves for pressure/voids ratio (as in Fig. 72) and time/consolidation may be drawn.

Gilboy has found that if the percentage consolidation be plotted against the square root of the time to a logarithmic scale, the upper portion of the curve becomes a straight line, as shown in Fig. 72.

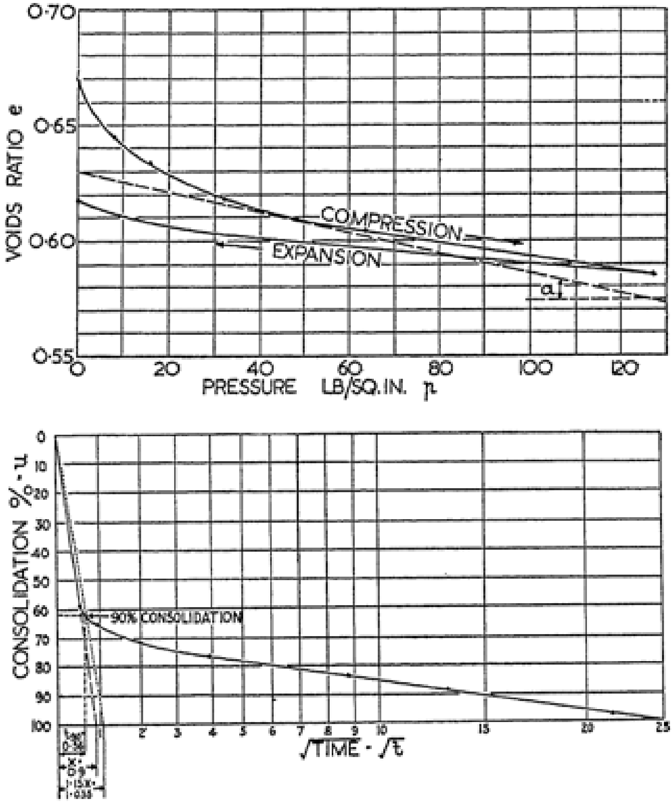


FIG. 72.—p-e and Percentage Consolidation–$\sqrt{t}$ Curves. (Problem 84.)

The theoretical zero lies on the vertical axis, and the straight line portion of the graph is produced to cut the horizontal axis at a distance X, as shown. A second straight line is drawn through the theoretical zero to an intercept on the horizontal axis equal to a constant proportion of 1·15X, and this line cuts the curve at a point representing 90 per cent. theoretical consolidation. 90 per cent. consolidation is adopted empirically for calculating the coefficient of consolidation, C. In this problem $t_{90} = 0{\cdot}56$ minutes from inspection of the lower diagram in Fig. 72.

The coefficient of consolidation, C (from Equation 50)

$$= \frac{T_{90} \times H_0^2}{t_{90}}$$ where H_0 is the half thickness of the clay for free drainage both sides. (Value of T obtained from Table 10.)

$$= \frac{0{\cdot}848\ H_0^2}{t_{90}}$$

$$= \frac{0{\cdot}848 \times 0{\cdot}974^2}{0{\cdot}56} = 1{\cdot}43 \text{ sq. cms./min.}$$

The coefficient of compressibility, a, is the slope of the p–e curve,

$$= \frac{e_1 - e_2}{p}$$

$$= \frac{0{\cdot}628 - 0{\cdot}572}{125}$$

$$= 0{\cdot}000448$$

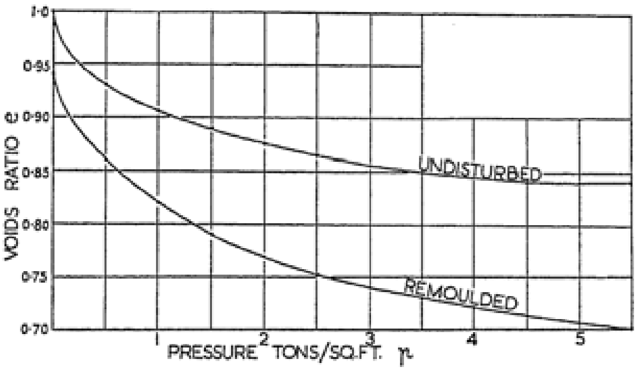


FIG. 73.—p–e Curves for Undisturbed and Remoulded Clay. (Problem 85.)

Problem 85. Fig. 73 gives p–e curves for a clay which exists in a layer 20 ft. thick at some depth below ground level. The load due to the over-burden is 2 tons per sq. ft., and a building is to be constructed which will increase this load to 3 tons per sq. ft. What total settlement can be expected due to the consolidation of this layer, provided (1) the building rests on spread footings constructed without disturbing the clay layer, and (2) the building rests on piles driven into the clay and thereby remoulding it?

(1) The voids ratio, e_1, for the undisturbed clay but with over-burden pressure of 2 tons/sq. ft. is 0·875 (from p–e curve in Fig. 73).

Hence, $$H_0 = \frac{20}{1 + 0{\cdot}875} = 10{\cdot}7 \text{ ft.}$$

If the pressure is increased to 3 tons per sq. ft., the voids ratio, e_2, is 0·855.

Therefore, $H = 10·7\ (1 + 0·955)$ or $H_0\ (1 + e)$
$= 19·85$ ft.

Hence the settlement to be expected $= 20 - 19·85 = 0·15$ ft. $= 1\frac{3}{4}$ ins.

(2) If the clay is disturbed by pile-driving, then H_0 will be 10·7 ft. as before, but e_2 for the pressure of 3 tons/sq. ft. will be 0·74 (from Fig. 73).

Therefore $H = 10·7\ (1 + 0·74) = 18·62$ ft.

Hence the expected settlement will be $= 20 - 18·62 = 1·38$ ft. $= 1$ ft. $4\frac{1}{2}$ ins.

Problem 86. A building is constructed on a layer of compact silt over a 15-ft. layer of clay overlying an 18-ft. layer of silty clay with an underlying layer of dense sand. The period of construction is to be spread over twelve months. The silt and sand may be considered to be incompressible. Assume the average pressure due to the building is 2,000 lb. per sq. ft. on the clay and 1,600 lb. per sq. ft. on the silty clay. Laboratory tests indicate the following average values :—

CLAY. $e = 1·12$. Slope of p–e curve, $a = 0·00002$ s. cm./gr.
$K = 0·0000004$ cm./min.

SILTY CLAY. $e = 1·30$. Slope of p–e curve, $a = 0·00006$ s. cm./gr.
$K = 0·0000003$ cm./min.

Plot the time/settlement curves for the building.

(*a*) Considering the clay stratum of thickness 15 ft., from the p–e curve, the slope $a = \dfrac{e_1 - e_2}{p_1 - p_2} = -\ 0·00002 = \dfrac{1·12 - e_2}{-\ 977}$

when $p_1 = 0$, $e_1 = 1·12$
and $p_2 = 2{,}000$ lb./sq. ft.
$= 977$ gr./sq. cm.

Hence, $e_2 = 1·10$ and the total settlement of the clay layer

$$= H\frac{(e_1 - e_2)}{(1 + e)}$$

$$= 15\frac{(1·12 - 1·10)}{(1 + 1·12)}$$

$$= 0·141 \text{ ft.}$$

$$C = \frac{K_0}{a} = \frac{0·0000004}{0·00002} = 0·002$$

$$H_0 = \frac{15}{1 + 1·12} = 7·08 \text{ ft.}$$

$$t = \frac{7·08^2 \times N}{1{,}400 \times 0·002} = 17·9\,N \text{ or } N = \frac{t}{17·9}$$

Hence, settlement ΔH at any time $= u \times$ (total settlement)

$$= 0{\cdot}141u$$

The value of u is ascertained from Table 11 for various values of N which are determined for a number of values for t. These values are given in Table 13(*a*).

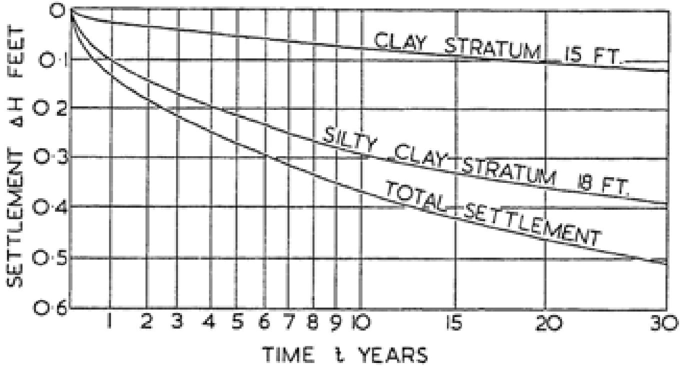


FIG. 74.—Time-Settlement Curves. (Problem 86.)

(*b*) Considering the silty clay stratum of thickness 18 ft., from the p–e curve, the slope

$$a = \frac{e_1 - e_2}{p_1 - p_2}$$

$$= \frac{1{\cdot}30 - e_2}{-782} = -0{\cdot}00006$$

when $p_1 = 0,\ e_1 = 1{\cdot}3$

and $p_2 = 1{,}600$ lb./sq. ft.

$$= 782 \text{ gr./sq. cm.}$$

$$e_2 = 1{\cdot}25$$

$$C = \frac{K_0}{a} = \frac{0{\cdot}0000003}{0{\cdot}00006} = 0{\cdot}005$$

$$H_0 = \frac{18}{1 + 1{\cdot}3} = 7{\cdot}85 \text{ ft.}$$

$$t = \frac{7{\cdot}85^2}{1400 \times 0{\cdot}005} N = 8{\cdot}75N$$

Total settlement of the silty clay layer

$$\Delta H = H\frac{(e_1 - e_2)}{(1 + e_1)}$$

$$= 18\frac{(1{\cdot}30 - 1{\cdot}25)}{(1 + 1{\cdot}30)}$$

$$= 0{\cdot}39 \text{ ft.}$$

TABLE 13(*a*).

t (years)	1/12	1/4	1/2	1	1·5	2	3
N	0·00466	0·014	0·028	0·056	0·084	0·112	0·168
u	0·0466	0·08	0·12	0·17	0·205	0·24	0·298
ΔH_t (feet)	0·0065	0·0115	0·0141	0·024	0·029	0·034	0·042
t (years)	4	5	7	10	15	20	30
N	0·224	0·28	0·39	0·56	0·84	1·12	1·38
u	0·3386	0·3786	0·45	0·535	0·65	0·7334	0·85
ΔH_t (feet)	0·048	0·0535	0·0635	0·0755	0·092	0·105	0·12

TABLE 13(*b*).

t (years)	1/12	1/4	1/2	1	1·5	2	3
N	0·0095	0·0285	0·057	0·114	0·171	0·23	0·345
u	0·065	0·12	0·171	0·243	0·30	0·35	0·425
ΔH_t (feet)	0·025	0·047	0·067	0·095	0·117	0·14	0·17
t (years)	4	5	7	10	15	20	30
N	0·46	0·57	0·8	1·14	1·71	2·3	3·45
u	0·485	0·54	0·635	0·739	0·85	0·915	1
ΔH_t (feet)	0·19	0·21	0·25	0·29	0·33	0·36	0·39

TABLE 13(*c*).

t (years)	1/12	1/4	1/2	1	1·5	2	3
ΔH_t (feet)	0·0315	0·0585	0·0811	0·119	0·146	0·174	0·212
t (years)	4	5	7	10	15	20	30
ΔH_t (feet)	0·238	0·2635	0·3135	0·3655	0·422	0·465	0·51

Hence, settlement ΔH at any time $= 0{\cdot}39u$.

The values in Table 13(*b*) can then be determined.

(*c*) The settlement/time curve for the building will be the accumulation of the above two curves. The total settlements will be as in Table 13(*c*).

The three curves plotted from the above results are shown in Fig. 74.

The Effects of Vibrations on Soils

In recent years a more complete investigation has been carried out on the resistance of soils to vibrationary or slow repetitional loading, and the value of such investigations is of much practical importance to engineers responsible for the design of machine foundations, roadway and aerodrome pavements and railways. Furthermore the compaction of soils by vibration and the seismic method of soil exploration are also connected with the principles involved.

Any structure founded on a cohesionless soil is likely to settle excessively if that soil is subject to vibrations due to moving machinery, traffic or pile-driving. Sands are found to be more susceptible to deformation by vibrationary loading than clays, and this is particularly so when the sand is uniformly graded. The settlement due to vibrationary loading is many times greater than that produced by an equivalent static load and its magnitude is dependent upon the frequency.

The greatest settlements are produced within a range of 300 to 2,000 cycles per minute, and within this range lie the natural frequencies of compressors, gas and diesel engines, with the result that the effect of the operation of these machines on foundation settlement is very noticeable. When the operational speed of a machine coincides with the natural frequency of the soil foundation, resonance will occur which can lead to structural damage.

The Oscillation Theory.

Vibrationary or slow repetitional loads dependent upon their point of application and direction can produce six types of displacement. One is in a vertical direction, two are in horizontal directions at right angles to each other, and there are three rotational displacements about the three axes. However, experiments have been largely confined to the simplest form of motion in a vertical direction, but it must be borne in mind that the more complicated types of motion can occur.

Considering the theory of free vibration, if a weight, W, is

supported by a spring of stiffness, K, and is depressed vertically and the pressure quickly released, the weight, W, will vibrate up and down in a vertical direction. The frequency of such free vibrations is given as follows:—

$$f = \tfrac{1}{2}\pi\sqrt{\frac{K}{M}} = \tfrac{1}{2}\pi\sqrt{Kg}/W.$$

Where, f = the natural frequency of free vibration,
K = the spring coefficient (lb. required to compress the spring 1 in.),
M = Mass,
W = weight and
g = acceleration due to gravity.

These vibrations will decrease in amplitude until they stop entirely and the frequency of such damped vibrations becomes:—

$$f_d = \tfrac{1}{2}\pi\sqrt{\frac{K}{M} - C^2/4M^2}$$

where, c = the damping constant.

In Germany in 1934, H. Lorenz during his investigations into the dynamics of soils considered a mass of soil below a foundation to act as a spring. Also, if the foundation vibrated it was reasoned that the soil below would vibrate with it, and thus the last equation becomes:—

$$f_n = \tfrac{1}{2}\pi\sqrt{\frac{LAg}{W + w}}$$

where, L = K/A = dynamic modulus of soil reaction,
W = weight of foundation system, and
w = weight of vibrating soil below the foundation.

It is difficult to assign a definite value to w, as the oscillations of a machine and foundation together with the soil mass below are somewhat complex and of the nature indicated by Messrs, J. H. A. Crockett and R. E. R. Hammond, in the *Journal of the Institution of Civil Engineers*, 1947, Paper, " Reduction of Ground Vibrations in Structure ". In Fig. 74*a* is indicated the way in which a foundation resting on a soil sets up a heaving action in the surrounding ground. When there is a downward impulse, there occurs an upstanding ring around the foundations, which alternates to a depression when the impulse is upwards.

The Natural Frequency of a Site.

In 1933 experiments were carried out by the German Research Society for Soil Mechanics (known as DEGEBO), using a simple oscillator consisting of two eccentrically supported discs revolving

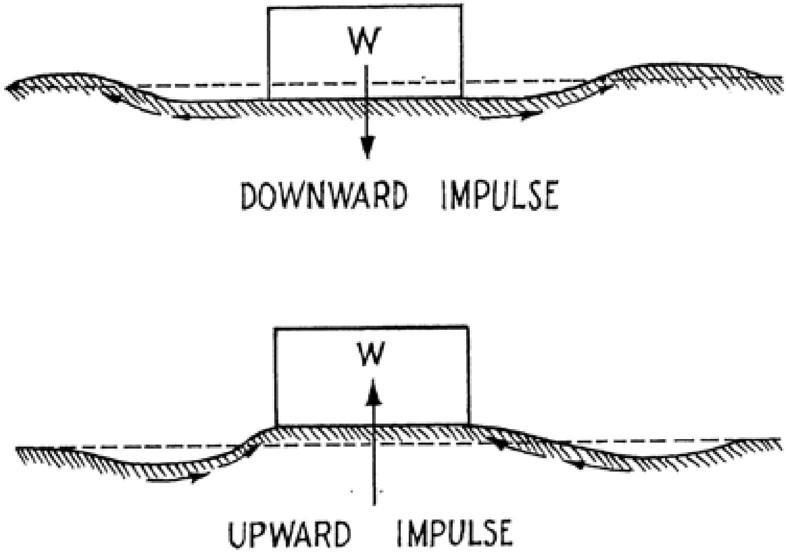


FIG. 74*a*.—Heaving action set up by a foundation.

in opposite directions. The eccentricity of the discs can be changed, thus altering the value of the centrifugal force, and by varying the speeds the frequency of the vibrations transmitted to the ground can be increased or decreased as required. The discs rotate in opposite directions and the horizontal forces are thereby balanced, resulting in the vertical forces being superimposed in the form of a sine curve as shown in Fig. 74*b*.

The experiments carried out by DEGEBO indicate that if a vertical impact is imposed upon the ground, the soil will oscillate vertically in the vicinity for several cycles. Furthermore the frequency of such oscillations is constant within limits for any particular type of soil or strata, and the higher the frequency, the greater will be the bearing capacity of the soil. The following table indicates the values of the natural frequencies of various soils as established by Lorenz in 1934, with a standard type oscillator, supplemented with values obtained by Andrews and Crockett in 1945, using hammers.

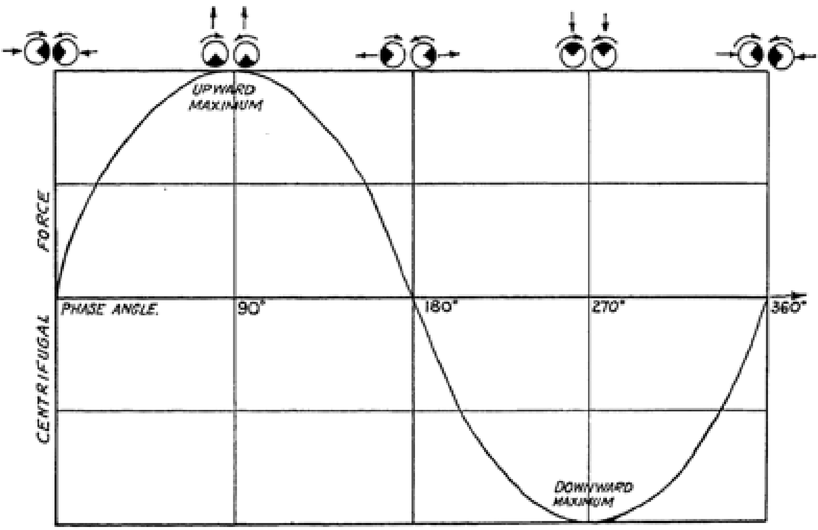


FIG. 74*b*.—Forces due to revolving of eccentric discs.

TABLE 13(*d*).

Natural Frequencies of Soils.

	Natural Frequency. c/s.	Ultimate Bearing Pressure. tons/sq. ft.	Ultimate Bearing Pressure. Kg/cm.^{2}
Lorenz, 1934. (Values measured using oscillator.)			
6 ft. peat overlying sand	12·5	—	—
Loose fill with medium sand	19·2	1·0	1·05
Dense compacted cinder fill	21·3	1·5	1·6
Dense compacted loamy sand fill	21·7	1·7	1·9
Moist lias clay	23·8	2·5	2·7
Dense uniform medium sand	24·1	3·0	3·25
Uniform coarse sand	26·2	4·0	4·35
Dense compact non-uniform sand	26·7	4·5	4·9
Dense pea gravel } Compact clay }	28·1	4·5	4·9
Limestone	30	—	—
Hard sandstone	34	—	—
Andrews and Crockett, 1945. (Values measured using hammers.)			
Waterlogged estuarine silt	10	0·75	0·8
Light soft clay	12	1·0	1·05
Light waterlogged sand	15	1·5	1·6
Medium clay	15	2·0	2·17
Layers of hard peat and sand mixed	17	2·0	2·17
Stiff clay	19	3·0	3·25
Limestone	30	—	—

The results of these experiments are limited, and it must not be assumed that the value of the natural frequency of a soil is a clearly defined physical property which can be related to its bearing capacity or other characteristics. G. P. Tschebotarioff draws the conclusion that the establishment of soil coefficients, which will enable the calculation of natural frequencies of soils to be made, is an indeterminate problem and it is necessary to employ semi-empirical methods for machinery foundation design. Furthermore, the degree of magnification of the dynamic forces at resonance depends upon damping factors which are also indeterminate.

Although there is no definite value for the natural frequency of a certain soil, it will be seen from the experimental results in Table 13(d) that there exist certain ranges of such frequencies. It has been ascertained by experiment that the natural frequency of a soil varies in accordance with the position and existence of buildings at the site investigated, and it is more correct, therefore, to refer to the natural frequency of the site rather than the soil.

With a view to relating the resistance of different types of soil to vibratory and slow repetitional loading, tests were carried out in Princeton University between the years 1943 and 1946 by Tschebotarioff and McAlpin with equipment of the California Bearing Ratio type. The experiments indicated that uniform sands were the most susceptible to vibrations and penetrations of the needle were between fifteen and seventy times greater than the penetration due to an equivalent static load. Well graded dry sands offered considerable resistance to penetration, which was between two and fifteen times greater than that obtained by an equivalent static load. Large penetrations of the plunger occurred with well graded sands under various conditions of saturation, and it is clear that extensive foundation settlement would occur with such sands if subjected to vibration.

With regard to clay soils, it was found that compacted clays are practically unaffected by vibrations, and the penetrations are equal to those obtained from static loading. Sand and clay mixtures when well compacted are similarly unaffected by vibrations, but saturated or loosely compacted sand-clay mixtures have a similar behaviour to that of graded sands.

It would seem that the shearing strength of sands, which is dependent upon external pressures, becomes reduced by vibrations, due to the individual sand grains slipping over each other. In the case of consolidated clays, the shearing strength is independent of external pressures and hence the cohesion between the particles is practically unaffected by vibration.

CHAPTER XII

FOUNDATIONS—SETTLEMENTS DUE TO CONSOLIDATION OF DEEP STRATA

Consolidation in Deep Strata

In the preceding chapter formulæ and tables were given which enable the settlement of any structure to be calculated. It is necessary, however, to give further consideration to settlement which will occur due to a soft compressible layer of soil situated at some depth below a structure.

Plans of any important structure should be accompanied by an estimate of the possible settlements which may be expected to occur at various points, their final value and rate of occurrence.

Settlements attributable to a deep soft layer of soil, such as a clay, will occur due to (*a*) the over-burden, (*b*) the clay stratum itself and (*c*) the load of the proposed structure.

(*a*) The settlement due to the over-burden will be dependent on the pressure distribution in the clay layer. If full consolidation of the clay has taken place, then the pressure distribution will be uniform, as indicated by the diagram ABCD, in Fig. 75, and for calculations the coefficients tabulated in Table 11, for the Case A, may be used. (See Chapter XI.)

If the over-burden is a recent deposit it may be that only 50 per cent. of the consolidation of the clay has taken place. In such an instance the triangle ABC, in Fig. 75, will represent the pressure area. The pressure will vary from a maximum at the upper surface of the clay layer to zero at the lower surface. The coefficients for Case C, in Table 11, will be applicable in this instance for the calculation of settlement which has taken place.

(*b*) Settlement due to the clay layer itself will vary from zero pressure at the upper surface to maximum pressure at the lower surface of the clay stratum. The pressure area will be represented by a triangle such as BCE, in Fig. 75, and the coefficients for Case B, in Table 11, will be used for calculations to ascertain settlements which have occurred.

(*c*) Settlement due to the load of the proposed structure will vary as the pressure from the upper to the lower surface, and the pressure area will be trapezoidal as indicated by FGHJ, in Fig. 75. In this case it is necessary to combine Cases A and C, in Table 11, in order to compute the settlement which will occur.

The simplest method of calculating the rate of settlement is

by the use of coefficients, which have been tabulated by Professor Kimball (Table 14). If the load at the top of the clay stratum is P_1 varying to P_2 at the bottom of the clay layer, then for trapezoidal pressure the value of

$$V = \frac{P_1}{P_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

The value of the constant N previously referred to in Chapter XI, may be determined by adding to the value of N ascertained from

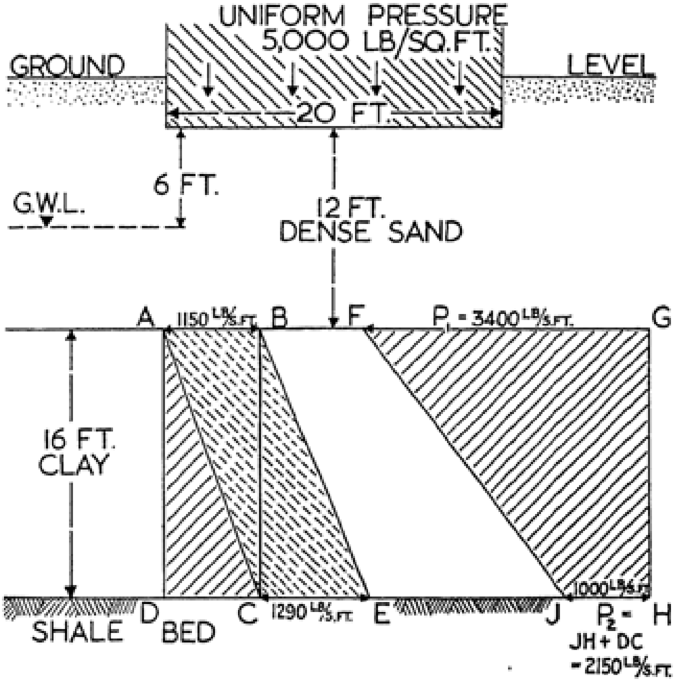


FIG. 75.—Pressure under Centre of Foundation. (Problem 87.)

Table 11, for Case A, J times the numerical difference between the N values for Cases A and B;—or J^1 times the numerical difference between the N values for Cases A and C. (It should be noted that the numerical difference is stated, and not the algebraical difference.)

For varying values of V, the coefficients in the form of a percentage for J and J^1 are set out in Table 14, and this table should be used in conjunction with the values given in Table 11.

The following problems will indicate the method adopted to

determine differential settlements due to the consolidation of deep clay strata.

Problem 87. Calculate the probable settlement in ten years under the centre and under one corner, of a foundation 40 ft. long and 20 ft. wide for a building exerting a uniform pressure of 5,000 lb. per sq. ft. Underlying the foundation is a stratum of dense sand 12 ft. thick, below which is a 16-ft. layer of clay over-

TABLE 14.

Influence Coefficients for Settlement Problems.

V.	J.	V.	J^1.
0·0	1·00	1·00	1·00
0·1	0·84	1·5	0·83
0·2	0·69	2·0	0·71
0·3	0·56	2·5	0·62
0·4	0·46	3·0	0·55
0·5	0·36	3·5	0·50
0·6	0·27	4·0	0·45
0·7	0·19	4·5	0·42
0·8	0·12	5·0	0·39
0·9	0·06	5·5	0·36
1·0	0·00	6·0	0·34
		7·0	0·30
		8·0	0·27
		9·0	0·25
		10·0	0·23
		12·0	0·20
		15·0	0·17
		20·0	0·13

lying a shale bed. Ground-water level exists at a depth of 6 ft. below the foundation. The weight of the sand above ground-water level is 110 lb. per cu. ft. and below this level, 70 lb. per cu. ft. Specific gravity of the soil grains is 2·7.

The coefficient of consolidation is 0·000383 sq. cm. per minute. The laboratory compression or p–e curve may be drawn from the following results in Table 15.

1. *Pressure distribution.*

(a) *Sand layer.* Considering 1 cu. ft. of the dense sand stratum, if n is the height of the voids, then

$$\left\{\begin{array}{l}\text{The weight}\\\text{of the so-}\\\text{lids above}\\\text{G.W.L.}\end{array}\right\} - \left\{\begin{array}{l}\text{Buoyancy}\\\text{or weight}\\\text{of water}\\\text{displaced}\end{array}\right\} + \left\{\begin{array}{l}\text{Weight of}\\\text{water}\\\text{filling the}\\\text{voids}\end{array}\right\} = \left\{\begin{array}{l}\text{Weight of}\\\text{solids}\\\text{below}\\\text{G.W.L.}\end{array}\right\}$$

Assuming :—

$$1 \text{ cu. ft.} = 28{,}320 \text{ c.c.}$$

$$1 \text{ lb.} = \frac{1000}{2{\cdot}205} \text{ grs.}$$

$1 \text{ ft.} = 30{\cdot}48$ cms., then by substitution

$$\left\{110 \times \frac{1000}{2{\cdot}205}\right\} - \left\{(28{,}320 - 30{\cdot}48^2 n) \times 1\right\} + \left\{30{\cdot}48^2 n \times 1\right\} = \left\{70 \times \frac{1000}{2{\cdot}205}\right\}$$

Hence n = 5·475 cms. = 2·155 ins.

Assuming 1 cu. ft. of water weighs 62·4 lb. then 2·155 ins. = 11·2 lb. Therefore, the weight of the sand and water on the clay stratum will be :—

Sand above ground-water level	= 6 ft. × 110 lb./sq. ft. = 660 lb.
Sand below ground-water level	= 6 ft. × 70 lb./sq. ft. = 420 lb.
Water	= 6 ft. × 11·2lb./sq. ft. = 67·2 lb.
	Total weight = 1,147·2 lb.
	or approximately, 1,150 lb./sq. ft.

It may be considered that only 50 per cent. of the total consolidation of the clay due to the sand over-burden has taken place, and in such a case the distribution of pressure will be as indicated by the area ADC, shown in Fig. 75, for the amount of future consolidation. (The area ABC representing the distribution of pressure for consolidation which has already taken place.)

(b) *Load due to new structure.* Considering the load from the building, a trapezoidal area of pressure will occur. Reference is made to Fig. 62 (Chapter IX), for the distribution of vertical pressure for a half width of b equal to 10 ft. :—

(i) Under the centre of the foundation :—

The pressure at a depth of 12 ft. = 0·68p
= 3,400 lb./sq. ft.
The pressure at a depth of 28 ft. = 0·20p
= 1,000 lb./sq. ft.
(as shown by the area FGHJ in Fig. 75).

(ii) Under one corner of the foundation :—

The pressure at a depth of 12 ft. = 0·40p
= 2,000 lb./sq. ft.
The pressure at a depth of 28 ft. = 0·15p
= 750 lb./sq. ft.

(c) *Clay layer.* Considering the clay stratum, it may be assumed that complete consolidation of the clay due to its own weight has taken place, and therefore e = 1·13 from the p–e curve in Fig. 76, which has been plotted from Table 15.

TABLE 15.

Load, kg./sq. cm.	Voids Ratio, e.	Notes.
0·000	2·04	
0·467	1·56	
0·862	1·43	
1·654	{ —	Percentage compression—Nil.
	1·28	100 per cent. compression.
3·237	1·13	
1·657	1·15	
0·866	1·18	
0·470	1·22	
0·032	1·27	

Hence, $e = \dfrac{n}{1 - n} = 1{\cdot}13$ and $n = 0{\cdot}53$ cm. also $(1 - n) = 0{\cdot}47$ cm.

Since the clay is saturated, 1 c.c. will weigh

$$(1 \times 1 \times 0{\cdot}47 \times 2{\cdot}7) - (1 \times 1 \times 0{\cdot}47 \times 1) + (1 \times 1 \times 0{\cdot}53 \times 1) = 1{\cdot}33 \text{ gr.}$$

Hence the weight per cu. ft.

$$= 1{\cdot}33 \times \frac{28{,}320}{1000} \times 2{\cdot}205 = 83 \text{ lb./cu. ft.}$$

and at a depth of 16 ft. the pressure will be 16 × 83 = 1,330 lb./sq. ft. or 0·665 kg./sq. cm.; but e = 1·13 corresponds to a pressure of 3 kg./sq. cm. and correcting for e = 1·45, n = 0·59 cms. and (1 − n) = 0·41 cm. as obtained from Fig. 76.

Hence,

$$(1 \times 1 \times 0{\cdot}41 \times 2{\cdot}7) - (1 \times 1 \times 0{\cdot}41 \times 1) + (1 \times 1 \times 0{\cdot}59 \times 1) = 1{\cdot}29 \text{ gr.}$$

$$\text{Therefore, the weight per cu. ft.} = 1{\cdot}29 \times \frac{28{,}320}{1000} \times 2{\cdot}205 = 80{\cdot}5 \text{ lb./cu.ft.}$$

and at a depth of 16 ft. the pressure will be 16 × 80·5 = 1,290 lb./sq. ft. or 0·645 kg./sq. cm., for which e = 1·48.

The pressure distribution will occur as shown by the area BCE in Fig. 75, varying from zero at the top of the clay layer to 1,290 lb./sq. ft. at the lower surface in contact with the shale bed.

2. *Consolidation.*

Future consolidation due to the sand layer will occur when the building is constructed, and this pressure will vary from zero at the upper surface of the clay to 1,150 lb./sq. ft. at the lower surface.

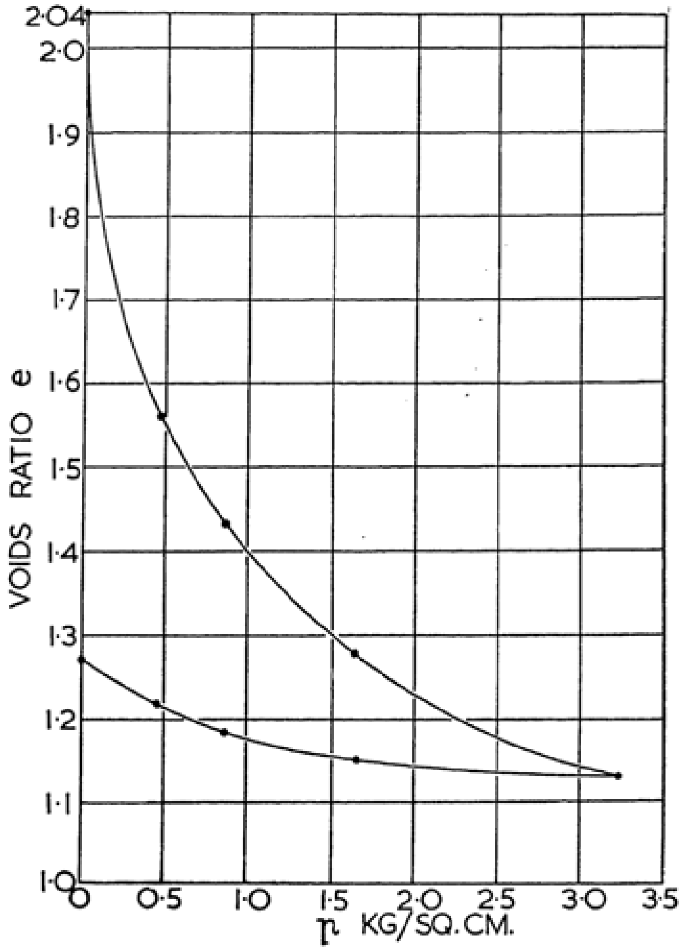


FIG. 76.—Laboratory Compression Curve. (Problem 87.)

When the load of the structure is applied :—

(i) Under the centre of the foundation :—

The pressure at a depth of 12 ft. = 3,400 lb./sq. ft.

The pressure at a depth of 28 ft. = (1,150 + 1,000) lb./sq. ft. = 2,150 lb./sq. ft.

(ii) Under one corner of the foundation :—
The pressure at a depth of 12 ft. = 2,000 lb./sq. ft.
The pressure at a depth of 28 ft. = (1,150 + 750) lb./sq. ft. = 1,900 lb./sq. ft.

The voids ratios from Fig. 76, before the construction of the building :—

at a depth of 12 ft. with a load of 0 lb./sq. ft. (= 0 kg./sq. cm.) $e_1 = 2{\cdot}04$;

at a depth of 28 ft. with a load of 1,150 lb./sq. ft. (= 0·575 kg./sq. cm.) $e_1 = 1{\cdot}53$.

Therefore, the average value for $e_1 = 1{\cdot}78$.
When the building has been constructed the voids ratios will be :—

(i) Under the centre of the foundation :—

At a depth of 12 ft., load = 3,400 lb./sq. ft.

$$= \frac{3400 \times 0{\cdot}4536}{30{\cdot}5 \times 30{\cdot}5} = 1{\cdot}66 \text{ kg./sq. cm.}$$

$$e_2 = 1{\cdot}28$$

At a depth of 28 ft., load = 2,150 lb./sq. ft.
= 1·05 kg./sq. cm.

$$e_2 = 1{\cdot}4$$

Therefore, the average value for $e_2 = 1{\cdot}34$

(ii) Under one corner of the foundation :—

At a depth of 12 ft., load = 2,000 lb./sq. ft.
= 0·975 kg./sq. cm.

$$e_2 = 1{\cdot}4$$

At a depth of 28 ft., load = 1,900 lb./sq. ft.
= 0·925 kg./sq. cm.

$$e_2 = 1{\cdot}41$$

Therefore, the average value for $e_2 = 1{\cdot}4$

Hence the total settlement ΔH under the centre of the foundation from equation (46)

$$= \frac{e_1 - e_2}{1 + e_1} H = \frac{1{\cdot}78 - 1{\cdot}34}{1 + 1{\cdot}78} 16 = 2{\cdot}53 \text{ ft.} = 2 \text{ ft. } 6\tfrac{1}{2} \text{ ins.}$$

and the total settlement ΔH under one corner of the foundation

$$= \frac{e_1 - e_2}{1 + e_1} H = \frac{1{\cdot}78 - 1{\cdot}4}{1 + 1{\cdot}78} 16 = 2{\cdot}19 \text{ ft.} = 2 \text{ ft. } 2\tfrac{1}{4} \text{ ins.}$$

3. *Rate of settlement.*

After the construction of the building :—

(i) Under the centre of the foundation :—

At a depth of 12 ft., the load $P_1 = 3{,}400$ lb./sq. ft.
At a depth of 28 ft., the load $P_2 = 2{,}150$ lb./sq. ft.

Therefore, from equation (52) $V = \frac{P_1}{P_2} = \frac{3400}{2150} = 1{\cdot}58$

Hence from Table 14, $J^1 = 0{\cdot}78$ (by interpolation).

(ii) Under one corner of the foundation :—

At a depth of 12 ft., the load $P_1 = 2{,}000$ lb./sq. ft.
At a depth of 28 ft., the load $P_2 = 1{,}900$ lb./sq. ft.

Therefore, $V = \frac{P_1}{P_2} = \frac{2000}{1900} = 1{\cdot}05$

and from Table 14, $J^1 = 0{\cdot}98$

The reduced value of the height of 16 ft. is obtained from the following equation,

$$H_0 = \frac{H}{1 + e_1} = \frac{16}{1 + 1{\cdot}78} = 5{\cdot}76 \text{ ft.}$$

Hence from equation (51)

$$t = \frac{N \times H^2}{1400 \times C}$$

$$= \frac{5{\cdot}76 \times 5{\cdot}76}{1400 \times 0{\cdot}000383} \times N$$

$$= 61\ N$$

If $t = 10$ years, then $N = \frac{10}{61} = 0{\cdot}164$.

Under the centre of the foundation, $N = J^1 (N_A - N_C) + N_A$
and $0{\cdot}164 = 0{\cdot}78 (N_A - N_C) + N_A$

Hence by trial and error from Table 11, $N_A = 0{\cdot}096$
$N_C = 0{\cdot}024$
$u\% = 22\%$

Therefore the settlement in 10 years will be $0{\cdot}22 \times 2{\cdot}53$ ft. $= 0{\cdot}56$ ft. $= 6\frac{3}{4}$ ins.

Under one corner of the foundation, $N = J^1 (N_A - N_C) + N_A$
and $0{\cdot}164 = 0{\cdot}98 (N_A - N_C) + N_A$

Hence by trial and error from Table 11, $N_A = 0{\cdot}096$
$N_C = 0{\cdot}024$
and percentage consolidation $u\% = 22\%$

Therefore the settlement in 10 years will be $0{\cdot}22 \times 2{\cdot}19$ ft. $= 0{\cdot}48$ ft. $= 5\frac{3}{4}$ ins.

Problem 88. Calculate the probable settlement in ten years under the centre and one corner of a foundation 200 ft. long and 100 ft. wide, for a building exerting a uniform pressure of 5,000 lb./sq. ft. Assume ground conditions are identical with those in the previous problem.

1. *Pressure distribution.*

Referring to Fig. 62 (Chapter IX) for the distribution of vertical pressure for a half width of b equal to 50 ft. :—

(i) Under the centre of the foundation :—

The pressure at a depth of 12 ft. $= 0{\cdot}95p$
$= 4{,}750$ lb./sq. ft.
The pressure at a depth of 18 ft. $= 0{\cdot}88p$
$= 4{,}400$ lb./sq. ft.

(ii) Under one corner of the foundation :—

The pressure at a depth of 12 ft. $= 0{\cdot}5p$
$= 2{,}500$ lb./sq. ft.
The pressure at a depth of 28 ft. $= 0{\cdot}45p$
$= 2{,}250$ lb./sq. ft.

2. *Consolidation.*

Future consolidation due to the sand layer will occur when the building is constructed and this pressure varies from zero at the upper surface to 1,150 lb./sq. ft. at the lower surface.

When the load of the structure is applied :—

(i) Under the centre of the foundation :—

The pressure at a depth of 12 ft. $= 4{,}750$ lb./sq. ft.
The pressure at a depth of 28 ft. $= (1{,}150 + 4{,}400)$ lb./sq.ft.
$= 5{,}550$ lb./sq. ft.

(ii) Under one corner of the foundation :—

The pressure at a depth of 12 ft. $= 2{,}500$ lb./sq. ft.
The pressure at a depth of 28 ft. $= (1{,}150 + 2{,}250)$ lb./sq.ft.
$= 3{,}400$ lb./sq. ft.

The voids ratios from Fig. 76 before the construction of the building are as in the previous problem, viz.,

At a depth of 12 ft., load = 0 lb./sq. ft. = 0 kg./sq. cm.
$e_1 = 2{\cdot}04$
At a depth of 28 ft., load = 1,150 lb./sq. ft. = 0·575 kg./sq. cm.
$e_1 = 1{\cdot}53$
Average value for $e_1 = 1{\cdot}78$

When the building has been constructed the voids ratios become :—

(i) Under the centre of the foundation :—

The pressure at a depth of 12 ft. = 4,750 lb./sq. ft.
= 2·32 kg./sq. cm.
$e_2 = 1{\cdot}18$
The pressure at a depth of 28 ft. = 5,550 lb./sq. ft.
= 2·70 kg./sq. cm.
$e_2 = 1{\cdot}155$
Average value for $e_2 = 1{\cdot}17$

(ii) Under one corner of the foundation :—

The pressure at a depth of 12 ft. = 2,500 lb./sq. ft.
= 1·22 kg./sq. cm.
$e_2 = 1{\cdot}36$
The pressure at a depth of 28 ft. = 2,400 lb./sq. ft.
= 1·66 kg./sq. cm.
$e_2 = 1{\cdot}28$
Average value for $e_2 = 1{\cdot}32$

Hence the total settlement ΔH under the centre of the foundation

$$= \frac{e_1 - e_2}{1 + e_1} H = \frac{1{\cdot}78 - 1{\cdot}17}{1 + 1{\cdot}78} 16 = 3{\cdot}87 \text{ ft.} = 3 \text{ ft. } 10\tfrac{1}{2} \text{ ins.}$$

and the total settlement ΔH under one corner of the foundation

$$= \frac{e_1 - e_2}{1 + e_1} H = \frac{1{\cdot}78 - 1{\cdot}32}{1 + 1{\cdot}78} 16 = 2{\cdot}65 \text{ ft.} = 2 \text{ ft. } 7\tfrac{3}{4} \text{ ins.}$$

3. *Rate of settlement.*

When the building has been constructed :—

(i) Under the centre of the foundation :—

The pressure at a depth of 12 ft. = P_1 = 4,750 lb./sq. ft.
The pressure at a depth of 28 ft. = P_2 = 5,550 lb./sq. ft.

Therefore, from equation (52) $V = \dfrac{P_1}{P_2} = \dfrac{4{,}750}{5{,}550} = 0{\cdot}856$

and from Table 14, $J = 0{\cdot}086$

(ii) Under one corner of the foundation :—

The pressure at a depth of 12 ft. $= P_1 = 2{,}500$ lb./sq. ft.
The pressure at a depth of 28 ft. $= P_2 = 3{,}400$ lb./sq. ft.

Therefore, from equation (52) $V = \frac{P_1}{P_2} = \frac{2{,}500}{3{,}400} = 0{\cdot}74$

and from Table 14, $J = 0{\cdot}162$

The reduced value of the height of 16 ft.

$$= H_0 = \frac{H}{1 + e_1} = \frac{16}{1 + 1{\cdot}78} = 5{\cdot}76 \text{ ft.}$$

Hence, from equation (51)

$$t = \frac{N \times H^2}{1400 \times C}$$

$$= \frac{5{\cdot}76^2}{1400 \times 0{\cdot}000383} \times N$$

$$= 61\ N$$

If $t = 10$ years, then $N = \frac{10}{61} = 0{\cdot}164$

(i) Under the centre of the foundation :—

$N = J\,(N_A - N_B) + N_A$
and $0{\cdot}098 = 0{\cdot}086\,(N_A - N_B) + N_A$
Hence, from Table 11, $N_A = 0{\cdot}08$
$N_B = 0{\cdot}25$
$u\% = 20\%$
and the settlement in 10 years will be $0{\cdot}20 \times 3{\cdot}87$ ft.
$= 0{\cdot}774$ ft. $= 9\frac{1}{4}$ ins.

(ii) Under one corner of the foundation :—

$N = J\,(N_A - N_B) + N_A$
and $0{\cdot}098 = 0{\cdot}162\,(N_A - N_B) + N_A$
Hence, from Table 11, $N_A = 0{\cdot}08$
$N_B = 0{\cdot}25$
$u\% = 20\%$
and the settlement in 10 years will be $0{\cdot}20 \times 2{\cdot}65$ ft.
$= 0{\cdot}53$ ft. $= 6\frac{1}{4}$ ins.

The comparison of the results of these two problems is interesting. It will be noticed that the additional width of the building in the second problem affects the underlying soil to a much greater depth, and the load areas of the clay stratum are increased in all cases by the additional width of the structure.

Although the loading of 5,000 lb./sq. ft. is the same in each case, under the centre of the building the total settlement of

2 ft. $6\frac{1}{2}$ ins. is increased to 3 ft. $10\frac{1}{2}$ ins. The differential settlements in Problem 87 are $4\frac{1}{4}$ ins., and in ten years 1 in., but in Problem 88 the settlements are increased to 1 ft. $2\frac{3}{4}$ ins. or, in ten years, 3 ins. It will be evident that considerable increase in settlement will occur with the increase in width of any structure although the foundation loading is not altered.

Note on the Consolidation of Clay Causing Hydrostatic Pressure

It will be readily appreciated that the loading of a clay stratum with a structure will cause consolidation, which will induce hydrostatic pressure in the water in the clay. This will result in a high fluid pressure on the walls and floor of a basement in contact with the clay.

Assume a building with a basement floor the foundation of which is 6 ft. below ground level, and carrying a load of 1 ton per sq. ft. If ground-water level occurs at 2 ft. 6 ins. below ground level, then the floor of the basement will be subjected to a pressure of water under a head of :—

$$\begin{aligned} & \frac{2240}{62{\cdot}5} + (6 \text{ ft. } 0 \text{ in.} - 2 \text{ ft. } 6 \text{ ins.}) \\ = \ & 35 \text{ ft. } 9\tfrac{1}{2} \text{ ins.} + 3 \text{ ft. } 6 \text{ ins.} \\ = \ & 39 \text{ ft. } 3\tfrac{1}{2} \text{ ins.} \end{aligned}$$

To avoid this condition it is necessary to provide a layer of pervious material such as hardcore and ashes between the clay and the basement floor. Stone drains should be constructed to lead away to a drainage system any water which is squeezed out of the clay into the filter layer. Such conditions are often the cause of seepage of water into basements, even when such basements have been tanked with asphalt. Within the experience of the authors, basement floors founded on clay have been forced upwards with hydrostatic pressure which has reached excessive proportions due to considerable consolidation of the clay.

Settlement on a Sand Foundation

The protracted settlements which take place with the consolidation of a clay or silt do not occur with sands. Sand is relatively incompressible, and permeability is so high that any consolidation will take place quickly, and will be small. When the load is applied, some settlement will naturally take place, but although with our present state of knowledge on this subject they cannot be calculated, they may be estimated from the results of the bearing test which will now be described.

Assuming ΔH = the settlement of a structure on a sand foundation,
q = the foundation pressure, and
B = the breadth of the foundation.

Then, $$\Delta H = q^2 \frac{B}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (53)$$

where N is a function of the depth to the centre of gravity of the stress area. The stress area will be typically as indicated in Fig. 77 (A), and approximately the value of $N = D + \frac{B}{2}$, where D is the depth of the foundation below ground level.

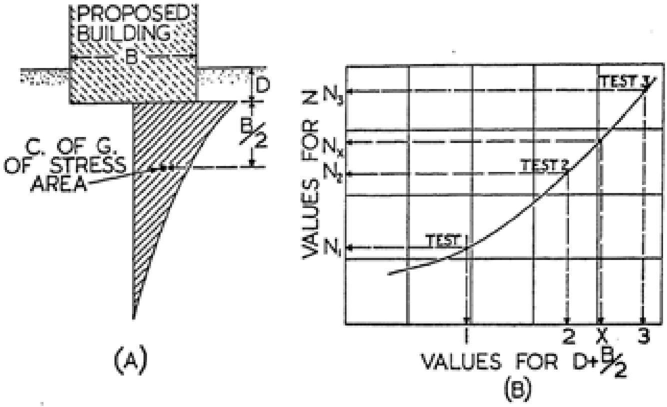


FIG. 77.—Settlements in Sands. Bearing Tests.

The value of N is determined by carrying out loading tests on small areas at three or more different depths. From these tests a curve is plotted for N and $D + \frac{B}{2}$ as in Fig. 77 (B), and from this graph a value for N_x can be obtained for any size foundation at any depth for this particular set of ground conditions. (More extensive details of this type of bearing test in connection with the construction of North German towns and harbours founded on sand may be found in *Die Bautechnik*, Volume 17, 1939, by Niebuhr.)

Notes on Piled Foundations

It is not intended in this volume to deal in detail with piled foundations, as this is an extensive subject which already has been adequately covered in a number of textbooks.* Piling is an alternative to plain footing foundations and it is felt that a brief outline of the essential features to be considered from a soil mechanics aspect would help to clarify the position.

Piling may be successfully used in the following three cases:—

* E.g. *Piling for Foundations* by R. R. Minikin (Crosby Lockwood).

(i) To transfer the load to a solid stratum. The piles are driven through a soft upper layer to a deeper stratum of rock, compact sand or stiff boulder clay. The piles act as columns and are known as "point-bearing". Friction between pile and soil is neglected.

(ii) To support the structure by the friction between the piles and the soil. If the soft upper layer of soil is of a thickness which exceeds the maximum constructional length for the pile, then the piles are driven as "friction piles" and they are designed to take full advantage of the friction between the pile and the soil.

(iii) To consolidate loose sandy soils. Compaction takes place if closely spaced piles are driven.

Piles are valueless in the following instances :—

(*a*) Clays cannot be consolidated by piling as in the case of sands. Clays are very slightly compressible at the impact and if such piles are driven, a heave of the ground surface usually occurs between the piles.

(*b*) Piles should not be driven deeply into a stiff clay without a study of the voids ratio of the clay having been made in its undisturbed state and remoulded condition, as such piles may break down the natural structure of the clay and weaken the strength of the soil.

(*c*) Piles should not be driven into a saturated fine sand deposit as they tend to decrease the voids of the soil to such an extent that serious settlement will develop.

(*d*) Piles are useless if driven into a stratum which will in turn load an underlying soft layer of soil, as consolidation of the soft stratum will eventually take place with the consequent extensive settlement of the structure and the piles together.

Piling Formulæ.

There are two groups of formulæ in use :—

(i) Static formulæ developed from conditions of equilibrium, and the value of the angle of internal friction of the soil.

(ii) Dynamic formulæ, which endeavour to determine the value of the pile resistance from the dynamic resistance.

Formulæ under (i) are not examined because their field of application is very narrow and results obtained through them are unreliable; there is no physical or other similarity between the capacity of two piles, one of which is placed by impact in its position whilst the other is considered free of impact.

There are a great number of formulæ under group (ii), but of these only the Hiley formula is extensively used in the British Isles. The *Engineering News* formula, which is mainly used in America, covers chiefly timber piles and is of the following form :—

$$\text{Driving Resistance in tons, } R = E \frac{Mh}{6(s + 1)}$$

The Hiley formula is :—

$$\text{Driving Resistance in tons, } R = E \frac{Mh}{s + 0 \cdot 5c}$$

where M = the height of hammer in tons,
h = the effective fall of the hammer in inches,
s = the final set in inches per blow,
c = the temporary elastic compression of the pile (inches),
and E = the efficiency of the blow.

The efficiency of the blow may be determined as

$$E = \frac{M + Pe^2}{M + P} \text{ if } M \geqslant Pe \text{ or}$$

$$E = \frac{M + Pe^2}{M + P} - \frac{(M - Pe)^2}{(M + P)^2} \text{ if } M < Pe$$

provided the weight of the hammer, M tons, is not less than $\frac{P}{3}$ tons. The weight of the pile P includes the weight of the helmet and dolly.

When piles are driven to refusal on rock, the driving stress can be found from R if a value of 0·5P is substituted for P. The driving stress should be ascertained in such cases to avoid damage through over-driving.

The temporary elastic compression of the pile is measured during driving as shown in Fig. 77*a*. A paper sheet is attached to the

Type of pile, packing, dolly, etc.	e
Reinforced-concrete pile driven with double-acting steam hammer on steel anvil	0·5
Reinforced-concrete pile driven with single-acting or drop hammer	0·4
Reinforced-concrete pile with timber cap to helmet or timber pile driven with single-acting or drop hammer	0·25
Head of pile or dolly in a deteriorated condition	Nil.

pile by drawing pins or elastic bands. A straight edge is supported by a light timber beam which is carried on two uprights fixed about 4 ft. away from the pile. A pencil is drawn across the

straight edge and a record of the form indicated is obtained as blows are delivered. Both the set, s, and the temporary elastic compression of the pile and ground, c, may be determined from

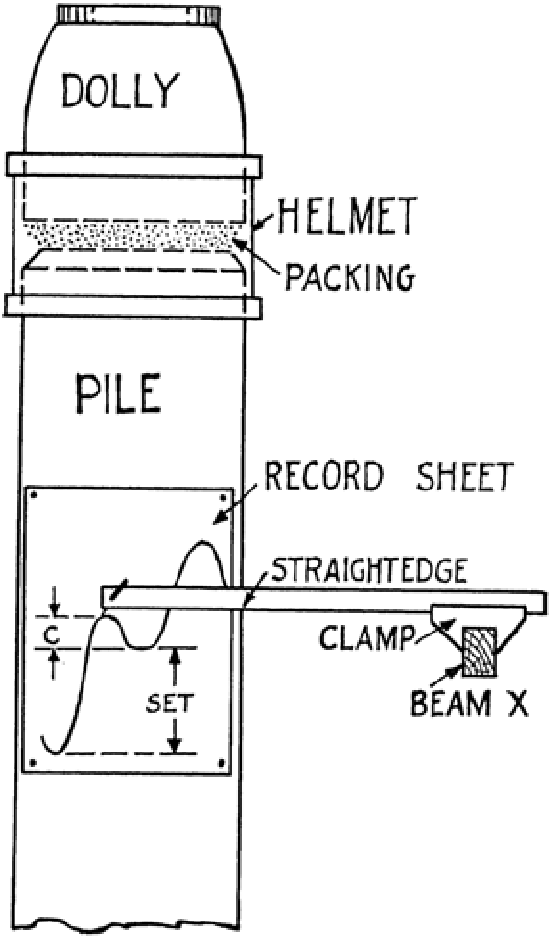


Fig. 77a.—Method of measuring Pile set and elastic compression of pile and ground.

this record. Hiley compiled tables for various values of c which varied as the severity of the driving, type of pile, etc., and these were published in his Paper presented to the Institution of Structural Engineers in 1930.

The coefficient of restitution and values for this factor are given in the Table on p. 172 :—

The effective fall of the hammer, h, is the free fall of a hammer released by a monkey trigger, and should not exceed 4 ft. 6 ins. For a drop hammer operated by a friction winch, h is equal to 0·8 times the free fall and for a single-acting steam hammer Mh is substituted by Wt + atm, where

W = Weight of moving ram (tons)
t = stroke (inches)
a = net area of the piston acted upon by steam (sq. in.)
m = mean steam pressure in the cylinder (tons per sq. in.).

When piles are driven to a batter, the adjustments shown in the following table should be made. In this case h is regarded as the length of travel of the hammer along the pile leaders.

Batter 1 in.	Per cent to be deducted from R.	Batter 1 in.	Per cent to be deducted from R.
12	1·0	6	3·00
11	1·25	5	4·00
10	1·50	4	5·50
9	1·75	3	8·50
8	2·00	2	14·00
7	2·50		

For the safe working load on a pile Hiley recommends a safety factor of 2.

Notwithstanding the vast amount of work already contributed towards establishing a piling formula which would be reliable, no other means, short of test loading, exists which would cover all circumstances.

Thus, during the driving of piles in clay, there is an impact of short duration on the clay at the foot. This produces a higher pressure on the clay than it will resist permanently, due to the fact that the pore-water cannot escape immediately, and this feature is known as the plasticity effect. Thus piles founded on clay carry much smaller loads for given sets and blows than similar piles in cohesionless soils.

Another effect in clay known as " take-up " occurs during the driving in clay which reduces the side friction or the adhesion of the pile and clay. After an interval, however, particularly if other piles are being driven nearby, the soil closes up and adhesion develops.

A new piling formula developed by Dr. O. Faber has been given in his Paper before the Institution of Civil Engineers in January, 1947. It is based on a large number of observations, but it is desirable for more extensive research to be undertaken before the formula can be adopted without reserve.

Groups of friction piles in clay will not carry a load equal to the load of one pile by the number of piles in the group. Load tests will provide the necessary data in such instances. Piles driven closely in clay cause the incompressible soil to heave between the piles and induce tensile stresses and cracks in adjacent piles. It is usual to drive alternate rows of piles in clay and complete the missing rows after some elapse of time. Friction piles should be spaced at a minimum distance of 3 ft. 6 in. centre to centre as the static resistance decreases rapidly at closer spacings. Point-bearing piles, however, may be as close together as driving conditions allow, usually up to a minimum of 2 ft. 6 in. centres.

Pile loading tests, previously referred to, may be made by means of applying a dead load either with kentledge or by jacking with hydraulic jack from two adjacent piles with a cross beam. Such tests should be made a month after driving in order that " take-up " or adhesion may be allowed to develop.

Friction piles are more useful in the case of narrow foundations, as in such instances the piles carry the pressure distribution deeper to the stronger layers of soil below. However, when the width of the foundation is much greater than the length of the piles, they do not effectively cause the pressure distribution to be shifted to a deeper layer and piles are not required in such a case.

Problem. A building of six floors with no basement is to be constructed on a site where the following soil conditions have been established from borings :—

12 ft. layer of fill material,
28 ft. layer of stiff clay, and
below 40 ft. of dense sand stratum.
Ground water level at a depth of 10 feet.

What type of foundation should be adopted and why?

Excavation in trench through the top layer of fill material to the stiff clay stratum would be more economically carried out than piling. The loads from such a building are not of a high order and the footings could be designed to be adequately supported by the stiff clay. (The properties of the clay should be determined by laboratory tests to support this method of design.) The ground water would not present difficulties as it could be pumped from the footing trenches easily, the head being only 2 ft.

The alternative to footing would be piling, which can be rejected for the following reasons :—

(*a*) Piles would be more expensive whether driven into the clay layer or through this to the sand layer.

(*b*) Piles driven into the clay layer would most likely cause the filling to consolidate and the clay stratum would be taking the full load of the structure in a similar manner to the proposal to adopt footings.

(*c*) Piles driven to the sand stratum would be doubtful, as skin friction would in any case transmit load to the clay, or water would reach the sand stratum along the sides of the piles with probable deterioration of its properties.

Problem. A pier for a bridge of several spans is to be founded through 20 ft. of water. Borings taken at the site indicate the following conditions:—

River bed to depth of 12 ft. of mud,
30 ft. layer of clay, and
below 42 feet dense sand layer.

What type of foundation and structure would be desirable?

The type of foundation necessary would be compressed air caissons sunk through the mud and clay to the sand bed. An open cofferdam would be too expensive owing to the depth of the water. Piles through the clay to the sand could not be used with caissons and it is necessary to sink the caissons themselves to the sand bed.

The layer of mud could be removed by dragline. The caissons, preferably of reinforced concrete, could be constructed on the shore and floated out to their positions. The clay stratum excavated down to the sand. Owing to the slenderness of the caissons the loading from the superstructure should be as nearly as possible near the centre of each pier. Arches should not be adopted, but either steelwork or reinforced concrete designed with centre bearings on the caissons.

The design should allow differential settlement without injury to the superstructure.

Problem. A reinforced concrete pile 14 ins. by 14 ins. and 40 ft. long is driven 30 ft. with a $3\frac{1}{2}$-ton drop hammer and 2 ft. 6 in. effective fall to a final set of $\frac{1}{2}$ in.

What is the safe load with a factor of safety of 1·5. A timber dolly is used with sawdust packing in the helmet. The temporary elastic compression is 0·8.

$$\text{Weight of pile} = \frac{14 \times 14}{144} \times 40 \times \frac{140}{2240} = 3{\cdot}4 \text{ tons.}$$

Weight of hammer = $3\frac{1}{2}$ tons, which is greater than the weight of the pile, P.

The coefficient of restitution for the conditions given is $e = 0{\cdot}85$.

Hence, $E = \dfrac{M + Pe^2}{M + P} = \dfrac{3{\cdot}5 + 3{\cdot}4 \times 0{\cdot}25^2}{3{\cdot}5 + 3{\cdot}4} = 0{\cdot}525$

and $\quad R = E \times \dfrac{Mh}{5 + 0{\cdot}5c} = 0{\cdot}525 \times \dfrac{3 \times (2 \times 12 + 6)}{0{\cdot}5 + 0{\cdot}5 \times 0{\cdot}8}$

$= 52{\cdot}5$ tons.

Therefore the safe load is $\dfrac{52{\cdot}5}{1{\cdot}5} = 35$ tons.

CHAPTER XIII

ARTIFICIAL CEMENTATION AND GROUND WATER LOWERING

Artificial Cementation

The shearing resistance of a granular soil may be developed if the individual grains are cemented together, thus increasing cohesion and decreasing permeability of the soil. Artificial methods of cementation have been devised which may be produced either by the injection of chemicals or cement grouting the soil.

Two methods of injection are in use :—

(i) The first type consists of injecting chemical fluids, which are pumped into the soil and allowed to coagulate or set in the interstices between the grains, thus converting the soil into a permanent artificial stone such as a sandstone.

(ii) The second type is the freezing process which is limited to saturated soils. Freezing can be maintained only temporarily while the excavation and completion of the foundations are in hand.

Injection processes are classified in the following Table 16 :—

TABLE 16.

Cementation Processes

SINGLE SOLUTION.			TWO SOLUTIONS.
Suspensions.	Emulsions.	Solutions.	
1. *Normal.* (*a*) Portland Cement. (*b*) Cement-clay Mixtures. 2. *Thixotropic.* Bentonite.	*Bituminous emulsions.*	*Single fluid.* Sodium silicate with a coagulator.	*Two fluids.* Successive injections of sodium silicate and an electrolyte.

In the case of the single fluid it is pumped into the soil, and it sets or forms a gel between each soil grain, whereas with two solutions they react to precipitate a cementing substance between the soil grains.

The choice between the various processes may be guided by the limits which are indicated on the grain-size chart in Fig. 78. Problems Nos. 2, 3 and 4 in Chapter I illustrated the mechanical analysis of soils necessary for the compilation of grain-size charts.

Single-Fluid Processes.

1. *Normal.*

(a) *Cement Grout.* Cement grout can be successfully employed in the treatment of coarse ballast or for sealing water-bearing fissures in rock, but will not penetrate medium and fine sands. It is a most valuable expedient for forming impermeable curtains beneath dams founded on fissured rock, and fissures over 0·1 mm. can be effectively sealed.

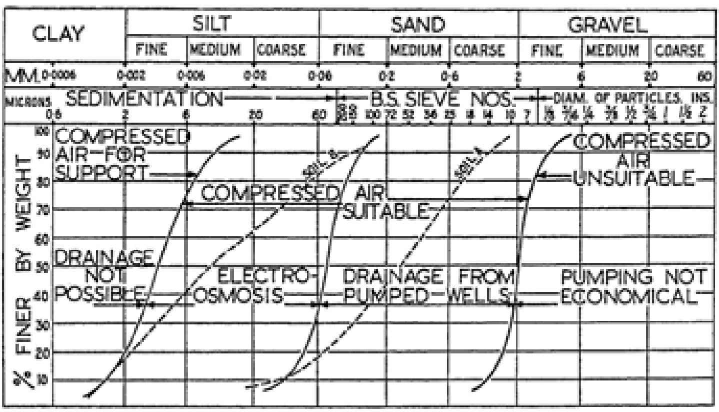


FIG. 78.—Limits for Artificial Cementation.

In cavernous ground cement is unsuitable and injections of hot asphalt have been successfully used.

Cement injections are most effective with very coarse sand and gravels, but should not be used with soils of effective grain size less than 1·4 mm. As previously stated, Fig. 78 indicates on a grain-size chart the limits between which cementation is effective. Injections of cement have been unsuccessful with medium and fine sands, as the grout forms a mass of cement at the end of the tube.

(b) *Cement-clay Mixtures.* In Germany a measure of success has been obtained with cement grouting in stiff-fissured clays, in particular at a site with side slopes for a canal cutting. In this case the grout forms a network of veins which prevent the percolation of water through the fissures and disintegration of the clay.

2. *Thixotropic Suspensions.* Thixotropy is the property of finely-divided solids in suspension setting to a gel if left undis-

turbed. Naturally occurring clays, known as Bentonites, form a gel when pumped into coarse sands, thus greatly increasing the soil strength and decreasing its permeability.

Bituminous Emulsions.

Bituminous emulsions can penetrate medium and fine sands with an effective size exceeding 0·1 mm., as indicated in Fig. 78. Immediately before injection a coagulator is added to the solution, which causes the emulsion to break down and flocculate, thus filling the pore spaces. The viscosity of the emulsion is considerably less than that of cement grout, and is not much more than that of water.

Bituminous emulsions are not used for very coarse sands or gravels, as the cost is much greater than cement grouting, and there is the possibility of rapid ground water flow removing the emulsion before flocculation takes place.

Chemical Consolidation. Single-Silicate Solution.

Several processes are available for the injection of sands down to an effective grain size of about 0·1 mm., and in all mixtures an essential ingredient is sodium silicate, together with other chemicals, which react to precipitate a silica-gel in the pore-spaces of the soil and transform the sand into an artificial sandstone.

Two-Fluid Processes.

Deep soil stabilisation may be accomplished by injecting two solutions separately, such as the Joosten or K.L.M. methods. When both chemicals come in contact they form a gel which on hardening becomes impervious to water and considerably increases the shearing strength of the soil.

(a) *The Joosten Process.* The first solution to be injected consists of sodium silicate or hydro-fluorosilicic acid, depending on the type of soil. Later a second solution of strong brine is injected, which has the instantaneous effect of precipitating a silica gel. In a good siliceous coarse sand an unconfined compressive stress up to 1,200 lb. per sq. in. is obtained, and in a fine sand a value of about 400 lb. per sq. in. is reached. It should be noted that this process cannot be used with clay soils.

The general method applied is to drive a perforated pipe into the soil 2 ft., then inject the first solution, drive the pipe a further 2 ft. and again inject the same solution, and so on until the desired depth is reached. The second solution is injected as each 2 ft. of the pipe is withdrawn. The extent of the penetration from the injection pipe depends on the liquid pressure used and the size of the soil voids.

(b) *The K.L.M. Method.* In this method two solutions, one of

which is sodium silicate, are injected together as a single mixture while the injection pipe is slowly withdrawn. The chemicals are modified according to the nature of the soil properties. Injection pipes are usually spaced at 20 ft. centres.

Single-fluid injections are much cheaper than two-fluid processes, but generally the single fluids, although decreasing the permeability of the soil, do not increase the shear strength to any marked degree. Single-fluid processes are chiefly used, therefore, in coarse sands and gravels for forming impermeable aprons beneath dams.

Two-fluid injections are more expensive, but the shear strength of the soil is considerably increased, and hence these processes are used for forming a "mat" of stabilised soil in under-pinning work and tunnel construction. With the use of compressed air in coarse sands and gravels the loss of air from the face during tunnel construction may be serious, and silicate injections put down ahead of the tunnel face are effective in preventing the loss of air, especially where the cover is small. The chemical consolidation of ballast in this manner was used in the construction of the Ilford Tube for the Eastern Extension of the Central London Railway. (See *The Ilford Tube*, by G. L. Groves, B.Sc., M.Inst.C.E., Paper read before the Institution of Civil Engineers, January 22nd, 1946.)

Freezing Processes

Freezing processes have the advantage that they are applicable to any water-bearing soil, whatever the permeability or grain size may be. The method is one which may be applied with success in dealing with silts under high hydrostatic pressure. The limits for the use of freezing processes are indicated on the grain-size chart in Fig. 78. Furthermore, the processes can be used to a much greater depth than can compressed air.

The main drawback lies in the high initial cost for the installation of the plant, and the length of time, often several months, required for freezing the ground. Below a depth of 100 ft. the freezing processes become an economical method.

Boreholes are sunk 3 ft. apart around the site of the excavation, and steel tubes 4 to 6 ins. diameter are inserted, closed at the lower end. Inside these tubes an inner tier of 2-ins. tubes are inserted with open ends, and both the inner and outer tier of tubes are connected to the brine delivery and return mains. Care must be taken to maintain vertical borings, which are easily obtained to a depth of 150 ft., but beyond this depth frequent checking is necessary. Any serious deflection of the bore-holes may mean discontinuity of the ice wall around the works.

The bore-holes should be taken down to a depth of at least 6 ft. in a clay stratum. The brine temperature is of the order of $-4°$ F.

One of the two following processes is employed :—

(a) *The Poetsch Process.* Ammonia is used as the refrigerant, and chilled calcium chloride brine is circulated through the tubes. A zone of frozen ground is formed around each tube, and these zones increase in size until they link up to form a continuous wall. Two or three months are required to form such a wall for a depth of 250 yds., and the shear strength of the soil is increased to such an extent that a compressive strength of 150 lb. per sq. in. is attained. The method is somewhat slow, and should the brine escape the resulting ground cannot be frozen.

(b) *The Dehottay Process.* In this process liquid carbon dioxide is circulated through the freezing tubes, and this method is somewhat cheaper and quicker than the previously described process. Generally, freezing processes are safest in avoiding subsidence, but settlements of several inches have taken place on certain occasions in the neighbourhood of construction works after the cessation of freezing. Such settlements may be attributable to the forming of ice lenses around the freezing pipes, and the remedy may lie in quicker freezing of the soil with lower temperatures.

The security of any excavation in frozen ground obviously depends on the continued operation of the refrigerating plant, and any breakdown extending over a period longer than two days may result in thawing and collapse of the excavations. Freezing of the pneumatic tools occurs, and relays of re-serviced tools must be available. The men employed must be provided with protective clothing.

Soil Stabilisation in Relation to Piling

Frictional resistance of piles can be increased by electro-osmosis, which has been recently applied in civil engineering for the drainage of fine-grained soils to wells. The flow of water is induced by an electric current applied through electrodes in the form of vertical rods and wells arranged alternately. This treatment causes the angle of internal friction of the soil to be increased, and water is expelled from the pores in the soil as if squeezed out by pressure. Subsequent immersion in water does not change the properties of this hardened soil.

This method was employed in the piled foundations of a bridge in Germany, where the load per pile was 90 tons. Ordinarily with this load and soil conditions the settlement of each pile varied from 3 ins. to 12 ins., but with identical conditions after electro-osmosis had been carried out the corresponding settlements amounted to a maximum of $\frac{1}{4}$ in. In this case the piles were sheathed with metal, which acted as the electrodes.

The quantity of current required increases constantly from zero to 1,500 coulombs of electricity per 12 gr. of water expelled, but after this value is reached the consumption of current increases rapidly and becomes uneconomical. The limits for soils which may be treated with this method are indicated on Fig. 79.

Electro-chemical Hardening of Clays

When an electric current is passed between aluminium electrodes inserted in a soft clay with a high moisture content, a decrease in the moisture content takes place in the vicinity of the anode,

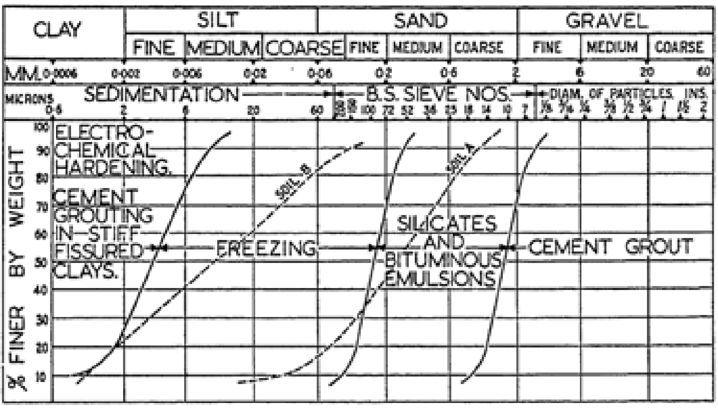


FIG. 79.—Limits for De-watering, etc.

and the clay hardens. This effect is similar to electro-osmosis, but other changes occur, and the liquid limit is reduced, as well as the capacity of the clay to absorb moisture. These changes are due chiefly to a chemical change, as the sodium and calcium ions of the clay particles are replaced by aluminium from the electrodes.

This method has been used successfully in Russia for shaft sinking through clays, and it has possibilities in surface hardening of clay slopes for the prevention of the percolation of water. The limits between which this process can be satisfactorily adopted is shown in Fig. 78.

Problem 89. The following are the results of a mechanical analysis of a sand deposit and a silty clay soil. Compile a grain-size chart and ascertain which of the following processes are suitable for these two soils :—

(*a*) Chemical consolidation.
(*b*) Cement grouting.
(*c*) Freezing.
(*d*) Bituminous emulsion.

TABLE 17.

Mechanical Analysis of Soils " A " and " B."

Particle size.	SOIL "A."		SOIL "B."	
	Per cent. retained.	Per cent. finer than	Per cent. retained.	Per cent. finer than
B.S. Sieve No. 12 .	3·4	96·6	—	—
" " 25 .	23·7	76·3	—	—
" " 100 .	63·7	36·3	9·1	90·9
0·05 to 0·005 mm. .	36·3	Silt	39·3	Silt
0·005 to 0·002 mm. .	—	—	29·2	Clay
Less than 0·002 mm. .	—	—	22·4	Clay

Grain-size curves for the two soils are indicated on the charts in Figs. 78 and 79.

From inspection it can be ascertained that Sample " A " is suitable for chemical consolidation with either silicates or bituminous emulsion. This soil may be excavated with the use of compressed air, or excavations may be drained by means of well pumping.

Sample " B " may be excavated by employing a freezing process or by adopting compressed air. Alternatively electro-osmosis may be used for drainage of the soil for excavation.

Ground-Water Lowering

In order to carry out construction works below ground-water level, it is possible to pump the water from wells and excavate the soil in a comparatively moist condition. In order to prevent piping, it is essential to pump water from points outside the proposed excavation, and not from sumps sunk in or ahead of the excavation. Therefore, the principle of ground-water lowering is to sink a number of wells around the perimeter of the excavation and to pump from them simultaneously. Within the ring of wells a continuous depression of the ground water occurs.

The problem which faces the engineer is how much water must be pumped to lower the water table sufficiently, and to do this it is necessary to determine the permeable qualities of the soils in the underlying strata.

Three methods for lowering ground water are in use :—

(a) *The Shallow-Well System,* in which bored filter wells approximately 6 to 12 ins. in diameter are sunk at intervals varying between 30 ft. and 50 ft. The pumps are situated on the surface, and the depth of the well is limited to 18 ft. The capacity of a well is largely dependent on the permeability of the ground in the

immediate vicinity. Each well is surrounded by a graded gravel filter, which prevents clogging of the wire screen surrounding the well and increases the permeability of the area adjacent to the well. Details of graded sand filters will be given in the next chapter.

This system is particularly suited to sites where headroom is restricted or where it is essential not to disturb the existing ground.

(b) *The Deep-Well System* consists similarly of bored wells at least 15 ins. in diameter, with submersible pumps and placed at greater intervals.

(c) *The Well-Point System* consists of 2-ins.-diameter pipes sunk by jetting and connected by means of a 6-ins.-diameter ring main to centrifugal and vacuum pumps sited at the surface.

Each well consists of a mild steel pipe about 25 to 30 ft. long. At the lower end for a distance of 3 to 4 ft. there are perforations or flutes to allow the flow of water, and a fine wire gauge filter is provided to exclude sand. The wells are placed at 4 to 6 ft. centres.

The system can be installed quickly, and is particularly suited to trench work. In one particular case dry sand for a depth of 6 ft. was obtained after pumping for forty-eight hours.

The permeability of a soil may be ascertained by one of the following methods :—

(1) *Pumping Test.* Curves are plotted denoting the level of the ground water during pumping from a bore-hole. From quantities of water pumped the effective permeability of the over-burden can be calculated, provided the effect of rainfall, etc., is taken into account. (See Problem 90.) Empirical formulæ can be used instead of plotting graphs, but the result is not so dependable.

(2) *Electrical method.* Perforated pipe-wells are sunk 30 to 40 ft. apart, containing electrodes connected by an electric circuit to a number of dry-cell batteries. An electrolyte is introduced to the upstream well, and when the electrolyte reaches the downstream well there is a rise in the current. The time is recorded, and from this data the velocity of flow and permeability of the soil can be ascertained.

(3) *Pressure Pumping.* A number of wells are sunk, a pumping connection made to one of the wells and water pumped into the well at a known rate and pressure. The rise in the ground-water table in the other wells gives sufficient information for the permeability of the soil to be determined.

(4) *Bore-hole Samples.* Undisturbed samples are taken from a bore-hole, and the average effective permeability of each stratum is obtained either by using Slichter's formula or laboratory tests. Details of the laboratory tests were included in Chapter II.

Professor C. S. Slichter's formula is as follows :—

$$\text{Permeability coefficient, kp} = \frac{1440 \times 0{\cdot}2012d^2}{uK'} \quad . \quad (54)$$

where d denotes the mean diameter of the soil grains measured in mm. u denotes the coefficient of viscosity of water (at 50° F., u has a value of 0·013077). K′ denotes a constant which depends on the porosity of the soil, and values for this constant may be obtained from the following Table 18.

TABLE 18.

Values of K′ for Percentage Porosity.

Per cent. Porosity.	Reciprocal of K′.	Per cent. Porosity.	Reciprocal of K′.	Per cent. Porosity.	Reciprocal of K′.
26	0·01187	34	0·02878	41	0·05339
27	0·01350	35	0·03163	42	0·05789
28	0·01517	36	0·03473	43	0·06267
29	0·01694	37	0·03808	44	0·06776
30	0·01905	38	0·04154	45	0·07295
31	0·02122	39	0·04524	46	0·07838
32	0·02356	40	0·04922	47	0·08455
33	0·02601				

The mean diameter of the soil grains is that size for which, if it were the diameter of all grains of the soil, the permeability of that material would be the same as that of the sample itself. Thus laboratory tests are necessary to establish the grain sizes of the various soils under consideration. Furthermore, it is necessary to compute the porosity from undisturbed samples.

Reference to Chapter II will indicate the methods of performing tests with a variable-head permeameter. For these tests the soil is packed by hand, and the samples are made with four different porosities, which are obtained by varying the degree of packing. Permeability is plotted against porosity to give a curve from which the permeability is obtained for the natural porosity of the soil.

After the permeability of each stratum has been ascertained, the total volumes in each layer must be determined, in order to compute an effective permeability, K, for the whole area, and where the deposits are continuous the following formula may be used :—

$$K = \frac{(kp)_1V_1 + (kp)_2V_2 \; . \; . \; . \; . + (kp)_nV_n}{V_1 + V_2 + \; . \; . \; . \; . + V_n} \quad . \quad (55)$$

A study of the entire area must be made and corrections introduced where water-bearing strata are insulated from adjacent strata, or inclined strata, causing a loss of head through the series of layers due to varying permeabilities.

The estimation of the effective permeability of a foundation soil enables the determination to be made for the amount of pumping, pumping plant and time required to lower the ground water sufficient to allow excavations to progress.

The quantity of water flowing to the pumps may be calculated from

$$q = kp \times As \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

where

kp denotes the permeability coefficient,
A denotes the area across the pumping limits, and
s denotes the slope of the ground-water table.

Problem 90. At a site where de-watering is required for the construction of foundations for a bridge a number of bore-holes are put down and a pumping test carried out. From readings taken in the various bore-holes, the ground-water tables before and after the test over the period of a week are plotted as in Fig. 80.

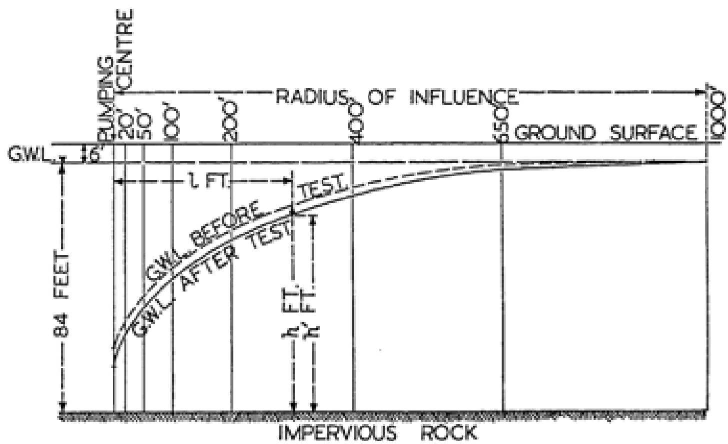


FIG. 80.—Ground-water Lowering. (Problem 90.)

An underlying impervious rock stratum exists at a depth of 90 ft. The original ground-water table is 6 ft. below ground surface and is level, hence there is no flow of ground water. The total amount of water pumped daily is 289,603 cu. ft. A 25 per cent. allowance should be made for voids after de-watering.

Calculate the permeability of the area, average areas, slopes and flows sufficient for the necessary equipment to be supplied for ground-water lowering.

From inspection of the curves in Fig. 80 it will be apparent that the radius of influence during the test is 1,000 ft. Several con-

TABLE 19.

Problem 90—Water to be Pumped due to Depletion of Storage and Flow due to Infiltration.

Distance from pumping centre, r_x ft.	Area between curves, A sq. ft.	Distance from pumping centre to c. of g. of area between curves, d ft.	Volume de-watered daily. $\frac{2\pi \times Ad.}{7}$ cu. ft.	Daily storage depletion allowing 25% for void. T = $\frac{2\pi \times Ad.}{7} \times 25\%$ cu. ft.	Storage depletion. Daily flow to pumps. D = Total—T = 208988—T cu. ft.	Daily flow due to infiltration. i = $P\left(1 - \frac{r_x^2}{r^2}\right)$ cu. ft.	Total daily flow to pumps. i + D cu. ft.	Average total daily flow to pumps, q between distances for r_x, cu. ft.
0	—	—	—	—	208,988	80,615	289,603	—
20	130	10	1,169	292	208,696	80,586	289,282	289,443
50	175	35	5,498	1,375	207,321	80,413	287,734	288,508
100	280	75	18,852	4,713	202,608	79,809	282,417	285,075
200	510	149	68,217	17,054	185,554	77,390	262,944	272,680
400	825	296	219,222	54,805	130,749	67,716	198,465	230,705
650	810	519	377,390	94,347	36,402	34,059	70,461	134,463
1,000	200	811	145,609	36,402	—	—	—	35,230

Total storage depletion 208,988 cu. ft.

Total water pumped = 289,603 cu. ft.

Hence, rate of infiltration, P = 80,615 cu. ft.

TABLE 20.

Problem 90—Average Areas, Slopes and Coefficients of Permeability.

Distance from pumping centre in feet.	Value of l feet.	Height h, ft. before test.	Area of cylindrical surface before test. $2\pi hl$, sq. ft.	Height h′, ft. after test.	Area of cylindrical surface after test. $2\pi h'l$, sq. ft.	Average area, sq. ft.	Slope of ground water curve, S.		Average value of slope.	Coefficient of Permeability between Sections $kp = q/As$.
							Before test.	After test.		
0– 20	10	27	1,697	22	1,382	1,540	0·55	0·5	0·53	354
20– 50	35	35	7,698	31	6,818	7,258	0·267	0·333	0·3	132
50– 100	75	44	20,737	40½	19,088	19,913	0·2	0·19	0·2	72
100– 200	150	55½	52,214	52	49,015	50,615	0·12	0·13	0·13	42
200– 400	300	69½	131,021	66	124,423	127,722	0·07	0·072	0·071	25
400– 650	525	80½	265,578	78	257,329	261,454	0·028	0·034	0·031	17
650–1,000	825	83½	432,889	82½	427,705	430,297	0·003	0·0086	0·0058	14

venient distances from the pumping centre are chosen, such as, 20 ft., 50 ft., 100 ft., 200 ft., 400 ft., 650 ft., and 1,000 ft.

The quantity of water, q, flowing past any chosen distance from the centre is made up of three parts :—

(i) The quantity flowing at the next outer distance,

(ii) The quantity of infiltration between the distance under consideration and the next outer distance, and

(iii) The quantity from depletion of storage between the distance under consideration and the next outer distance.

The volumes between the ground-water curves are estimated from Fig. 80. This is usually effected by taking off the area with a planimeter between each distance chosen and multiplying by twice the distance of the centre of gravity of the planimetered area from the pumping centre. Hence the Table 19 can be drawn up.

The average rate of infiltration is distributed in accordance with the following formula :—

$$i = P\left(1 - \frac{r_x^2}{r^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where

i denotes the portion of flow past any section that comes from the infiltration between that section and the radius of influence.

P denotes the portion of the total water pumped that is infiltration (= 80,615 cu. ft. in this problem),

r_x denotes the distance from the pumping centre to the section under consideration, and

r denotes the radius of influence (= 1,000 ft.).

It will be apparent from the above Table 19 that the rates of flow at each section is obtained by adding the flow due to infiltration to that of depletion of storage. The areas of the cylindrical surfaces are calculated as shown in Table 20, and the average areas determined. The slopes of each ground-water curve are estimated from Fig. 80 and averaged.

The permeability for each volume is calculated from Equation (56). This enables Table 20 to be compiled which together with Table 19, give all the essential data for determining equipment required for de-watering.

CHAPTER XIV

SOIL STABILISATION FOR ROADS AND AIRFIELDS

In recent years considerable research has been carried out on the distribution of wheel-loads from the surfacing to the formation by the introduction of a base layer of stabilised local material. The need for constructing roads and airfields in areas where good aggregates are not available has led to the development of stabilisation processes using low grade aggregates and soils.

A satisfactory base must withstand the action of traffic, water and frost. Mechanical compaction by rolling is an essential as the

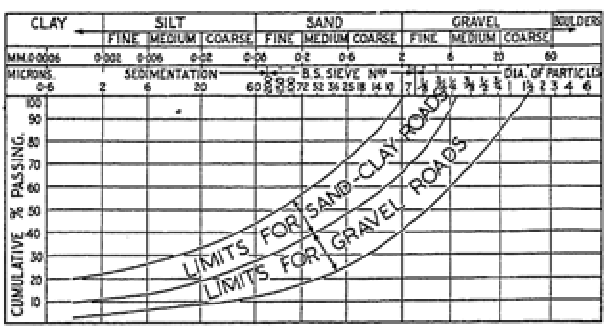


FIG. 81.—Grain Size Chart for Limits of Grading for Sand–clay and Gravel Roads.

physical properties of the soil is improved by increasing its shear strength, reducing its compressibility and decreasing its tendency to absorb water. The chief features relating to compaction were dealt with in Chapter V.

It is essential for a soil mixture which is to be compacted to be well graded, thus enabling the material to be rolled into a base containing the minimum of voids. The various limits of the grading for sand–clay and gravel roads are indicated by the curves in Fig. 81. When the particle-size distribution of the material lies outside these limits it is necessary to improve the mechanical properties by adding various stabilisers.

Methods of Soil Stabilisation.

Apart from mechanical compaction, soil stabilisation is obtained by the following processes :—

(1) Suitable grading of the local materials available which enables either one of the following methods to be adopted :—

(*a*) Water-bound macadam using stone aggregate only, which is a well-known method for road construction.

(*b*) Water-bound macadam using gravel, sand and clay in proportions which will give a high shear strength due to both cohesion and friction of the mixture. (See Fig. 37, Chapter V.) Roads of this material are common in South-east England, and are known as " Hoggin " roads.

(*c*) Clay-bound roads with the addition of deliquescent chemicals such as calcium or sodium chloride.

(2) Processes employing bituminous binders as stabilising agents, which consist of tar, bitumen emulsion, cut-back bitumen or asphaltic bitumen. The materials are graded, and vary from tar macadam to sand asphalts, with intervening types of clay-bound bases incorporating bitumen for waterproofing purposes.

(3) The use of Portland cement to produce soil-cement and lean-mix rolled concrete. This method is used chiefly with coarse-grained and sandy materials, although it is now being extended to fine-grained soils.

(4) The additions of resins, waxes, sulphite and lignin liquors to soils, which stabilise and impart waterproofing qualities to the mixture. The addition of wax causes a film action to take place, which greatly improves the waterproofing capacity of the stabiliser and, in a similar way, resins, such as sodium resinate and " vinsol " resin, restrict the absorption of water by the stabilised soil. Certain soils may be economically stabilised by the waste by-products from the paper industry, such as lignin liquors.

(5) Soils with a high clay content may be stabilised by heat treatment or by electro-chemical hardening as described in the previous chapter.

It is only possible in this volume to outline the main aspects of soil stabilisation, and those readers desiring further knowledge upon this subject are advised to study the following Papers :—

" The Use of Low-Grade Aggregates and Soils in the Construction of Bases for Roads and Aerodromes," A. H. D. Markwick and H. S. Keep. Road Paper No. 9. *Proc. of the Institution of Civil Engineers*, 1943.

" The Concrete Road : A Review of Present Day Knowledge and Practice with some reference to the Use of Stabilised Bases,"

F. N. Sparkes and A. F. Smith. Road Paper No. 17. *Proc. of the Institution of Civil Engineers*, 1945.

"Relationship of Runway Thickness and Under-Carriage Design to the Properties of the Sub-Grade Soil," H. Q. Golder. Airport Paper No. 4. *Proc. of the Institution of Civil Engineers*, 1946.

Sub-soil Drainage.

An important feature is the provision of adequate drainage of the sub-grade, and the primary function is to prevent water from entering the soil. The removal of water already in the soil is possible only with gravelly and sandy soils that drain freely. Silts and clays do not drain freely, as the pore water is mostly under capillary attraction and will not flow into an open drain except through cracks and fissures.

Drains should be constructed to intercept water likely to flow towards the roadway and to tap springs in the formation. Where ground-water level approaches formation level, drains will lower

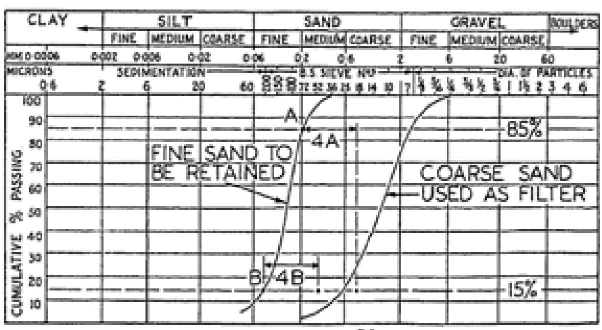


FIG. 82.—Grain Size Chart for Graded Filter Design.

the water table and prevent water tending to collect under the roadway.

Typical modern practice is to employ side drains placed near the edge of the road. These drains should be of the filter-drain type and communicate with a horizontal drain or permeable sub-base. The usual depth is about 3 ft., except where frost heave is likely, when depths up to 6 ft. are necessary. The life of these drains can be considerably prolonged by the use of a suitably graded filling. From a sizing analysis of the soil the coefficient of uniformity is measured, and a corresponding piping factor is found and used in choosing a suitable filter medium for the soil.

A filter of coarse sand allows water to flow through freely, but prevents fine sand from passing through, and holds back this

material. The grain-size of the filter must not be too large, or fine sand will be washed through the voids, nor must it be too small, or the filter material will cease to function. In the design of a graded filter Terzaghi's rule can be applied as illustrated in the grain-size chart in Fig. 82, which indicates the limits of the filter in terms of the sand to be retained. The filter material at 15 per cent. must not be finer than four times the grain size at 15 per cent. nor coarser than four times the grain size at 85 per cent. of the retained sand.

Flexible and Rigid Surfacings.

The objectives in the construction of surfacings are :—

(*a*) To construct a sub-grade of maximum strength.
(*b*) To protect the sub-grade by adequate drainage.
(*c*) To prevent the percolation of water into the sub-grade.
(*d*) To provide a surfacing of just sufficient thickness to protect the sub-grade from deformation.

Usually concrete surfacings are regarded as " rigid " surfacings, and macadam or aggregates with a cementing medium regarded as " flexible " surfacings.

The engineer is faced with the problem of designing the most economical form of surfacing, given the soil conditions at the site, the wheel loading and the intensity of traffic. Experience has shown that the design of a road or airfield surfacing is largely dependent on the properties and condition of the soil.

The action of traffic on surfacings is such that both the surfacing layer and the sub-grade deflect under the load and the deflection returns almost, but not quite, to zero. However minute the permanent deflection may be, changes occur in the sub-grade with the passage of each wheel. Weak roads usually fail progressively, and in the case of concrete slabs the gradual withdrawal of support of the sub-grade at the corners and transverse joints causes these portions of the slab to be most highly stressed. Eventually the slabs crack and, as is commonly observed, the cracks occur at the corners or a few feet from the transverse joints. For the prevention of sub-grade failure the soil must be sufficiently strong to resist plastic flow and sufficiently dense to avoid compaction by traffic.

Design of Flexible Surfacings.

A method of design which has gained considerable prominence is that known as the California Bearing Ratio, which is purely empirical, and is based on experience gained in California with roadways. The thickness of the surfacing required is obtained

from curves relating thickness to the California Bearing Ratio of the sub-grade for a given applied loading.

The test is made on material passing a $\frac{3}{4}$-in. sieve and the material retained by the sieve is replaced with an equal weight of soil which will pass a $\frac{3}{4}$-in. sieve but will be retained on a $\frac{3}{16}$-in. sieve. The optimum moisture content of the sample is obtained by a preliminary test in the manner mentioned in Chapter V.

The soil is compacted at optimum moisture content in a 6-in.-diameter cylinder in five equal layers, using fifty-five blows per layer with a 10-lb. rammer dropped through 18 ins. The top is trimmed, and the whole sample together with the cylinder is immersed in water for a period of four days under a surcharge equal to the weight of the proposed surfacing. The mould is removed and placed in a testing machine, in which a plunger 1·9 ins. in diameter is forced into the soil at a constant rate of 0·1 in. per minute. A diagram of the apparatus is given in Fig. 83. The dial gauge is graduated to record thousandths of an inch penetration up to a maximum of 0·5 in.

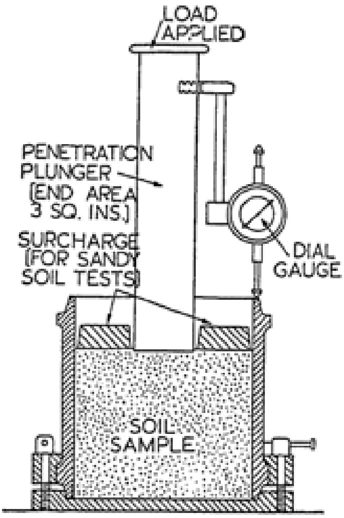


FIG. 83.—Apparatus for the California Bearing Ratio Test.

The California Bearing Ratio of the soil is obtained by dividing the pressure required to obtain 0·1 in. penetration by 1,000 and expressing the result as a percentage. The figure of 1,000 lb. per sq. in. is adopted as the pressure required for a first-class sub-grade formation of granular compact material.

The method has been extended to airfield runway design, and in practice it has been found to give good results. After the California Bearing Ratio of the soil has been found from the test, the necessary thickness of the surfacing is ascertained from the graph in Fig. 84, which includes both roadway and runway design. The curves in general are for use with a pressure of about 60 lb. per sq. in., but for high pressures exceeding 100 lb. per sq. in. an empirical addition of 20 per cent. should be made. The immersion period is an arbitrary one, dependent on climatic conditions, which must be considered for each case. The graph in Fig. 84 would be applicable to Great Britain, but in Australia the thicknesses given would probably be halved.

Design of Rigid Surfacings.

The knowledge of the California Bearing Ratio enables the design of concrete pavements for roads or airfields to be made. Fig. 85 gives such design curves based on the Westergaard formulæ

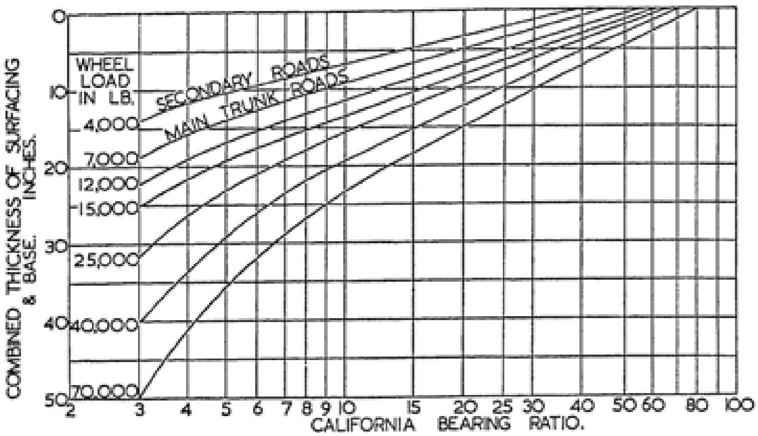


FIG. 84.—Design Curves for Flexible Surfacings.

using a working stress modulus of 350 lb. per sq. in. and a modulus of elasticity of 4,000,000 lb. per sq. in. The details of Westergaard's theory and analysis may be obtained from his Paper, "Stresses in Concrete Runways of Airports," by H. M. Westergaard, *Proceedings of the Highway Research Board, Washington*, Vol. 19, 1939 (pages 197–202).

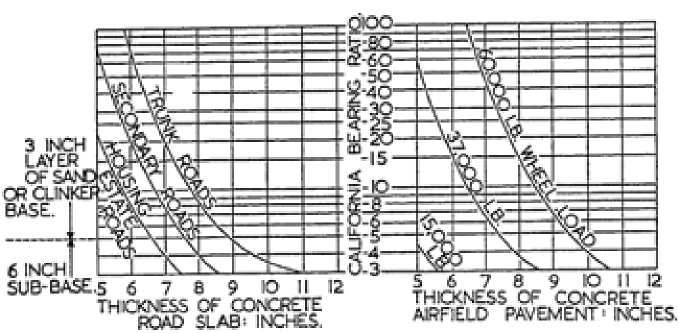


FIG. 85.—Design Curves for Rigid Surfacings.

Two cases of loading are considered, and Westergaard's formulæ are as follows :—

(1) Centre loading in which the stress in the concrete is

$$f_c = 0{\cdot}275(1 + \mu)\frac{P}{h^2}\log_{10}\frac{Eh^3}{kb^4} \quad . \quad . \quad . \quad (58)$$

when $a > 1{\cdot}724h$, then $b = a$,
when $a < 1{\cdot}724h$, then $b = \sqrt{1{\cdot}6a^2 + h^2} - 0{\cdot}675h$.

(2) Corner loading in which the stress is given by the following :

$$f_c = \frac{3P}{h^2}\left[1 - \left(\frac{Eh^3}{12(1-\mu^2)k}\right)^{-0{\cdot}15} \times a_1^{0{\cdot}6}\right] \quad . \quad . \quad (59)$$

where P denotes the wheel load in lb.,
h denotes thickness of slab in inches,
E denotes Young's modulus in lb. per sq. in.,
k denotes modulus of sub-grade reaction in lb. per sq. in. per in.
a denotes radius of contact area in inches,
a_1 denotes $a\sqrt{2}$, and
μ denotes Poisson's ratio.

The required thickness of concrete pavement may be obtained from Fig. 85, and this thickness may be reduced by 1 in. for each 5 ins. of good base material between the sub-grade and the slab, provided the slab thickness is a minimum of 6 ins.

The design curves indicate the value of soil compaction in pavement construction. Suppose, for example, that by compaction the California Bearing Ratio of a soil is increased from 3 to 6, then the thickness of pavement for an airfield to carry a 12,000-lb. wheel load would be reduced from 22 ins. to 15 ins., which is a saving of about 33 per cent.

Problem 91. The grain-size chart for a sandy soil may be prepared from the following sizing analysis carried out in the laboratory :—

Cumulative per cent. passing,	90·2	B.S. Sieve No.	7.
,, ,, ,, ,,	83·1	,, ,, ,,	14.
,, ,, ,, ,,	69·3	,, ,, ,,	25.
,, ,, ,, ,,	56·4	,, ,, ,,	52.
,, ,, ,, ,,	42·1	,, ,, ,,	100.
,, ,, ,, ,,	27·5	,, ,, ,,	200.

Would this soil be suitable for compacting by rolling into a base for the construction of a roadway ? What type of stabilisation of the soil could be adopted to produce a stable surfacing, if the sand is very dry ?

If the particle-size distribution curve for this sandy soil is plotted on the diagram given in Fig. 81, it will be found that the curve lies well within the limits for sand–clay roads, and therefore,

the soil can be satisfactorily rolled to compact it into a suitable base for road construction.

The sand is very dry, and with the addition of 2 per cent. of hydrated lime it is found possible to coat the sand grains with bitumen. In this instance it is recommended that this course be adopted to provide a good sand-asphalt surfacing to the roadway. (The construction of roads in Egypt and airfields in the British Isles has been extensively carried out during the last few years by means of this process with very successful results.)

Problem 92. The grain-size chart for a fine sand may be prepared from the following sizing analysis carried out in the laboratory :—

Cumulative per cent. passing,	94·5	B.S. Sieve No.	52.
,, ,, ,, ,,	84·9	,, ,, ,,	72.
,, ,, ,, ,,	69·2	,, ,, ,,	100.
,, ,, ,, ,,	30·4	,, ,, ,,	150.
,, ,, ,, ,,	15·1	,, ,, ,,	200.

Suggest a suitable type of sand blanketing which will allow free drainage of the soil and act as a filter to hold back the fine sand.

The grain-size chart in Fig. 82 indicates on the left the curve for the fine sand prepared from the laboratory analysis. From inspection of the curve at 15 per cent. passing, the retained sand is 0·07 mm., and, in accordance with Terzaghi's rule, the filter sand must not be finer than four times 0·07 mm., i.e. 0·28 mm.

At 85 per cent. passing the grain size of the retained sand is 0·2 mm., and hence the filter sand must not be coarser than four times this size, i.e. 0·8 mm.

A suitable coarse sand to act both as a blanket to allow free drainage and as a filter holding back the fine sand would be similar, therefore, to the following sizing analysis :—

Cumulative per cent. passing,	85	B.S. Sieve No.	7.
,, ,, ,, ,,	45	,, ,, ,,	14.
,, ,, ,, ,,	15	,, ,, ,,	25.

A particle-size distribution curve for this suggested filter material is shown on the right in the grain-size chart in Fig. 82.

Problem 93. In a California Bearing Ratio Test the pressure required to obtain 0·1 in. penetration at the standard rate in a soil sample is 9,500 lb. What thickness of roadway and base for a flexible surfacing would be required for heavy traffic roads of 7,000-lb. wheel load? If the sub-grade is compacted by rolling to a depth of 9 ins. and gives a California Bearing Test pressure

of 15,000 lb., what thickness of roadway then would be necessary?

The California Bearing Ratio of the soil in the first case

$$= \frac{9500}{1000} = 9{\cdot}5$$

Referring to the design chart in Fig. 84, the combined thickness of the road surfacing and base for the appropriate curve for main trunk roads (7,000 lb. wheel load) is 9·5 ins.

The California Bearing Ratio of the compacted sub-grade

$$= \frac{15{,}000}{1000} = 15{\cdot}0$$

Again with reference to Fig. 84, the road thickness necessary is ascertained to be 6 ins.

This illustrates how partial compaction of the sub-grade by rolling would enable a saving of 37 per cent. to be effected in the thickness of the road surfacing, which clearly indicates the economic value of compaction.

Problem 94. A pressure of 17,500 lb. is required to obtain standard penetration in a soil for the California Bearing Test. What thickness of concrete pavement would be required for an airfield to withstand wheel loads up to 37,000 lb?

$$\text{The California Bearing Ratio} = \frac{17{,}500}{1000} = 17{\cdot}5$$

From Fig. 85, right-hand diagram, the concrete pavement thickness for wheel loads up to 37,000 lb., and California Bearing Ratio of 17·5 is 5·85 ins.

Problem 95. Design a suitable concrete road section for a secondary road (4,000 lb. wheel load) to be constructed on a soil with a California Bearing Test load of 20,000 lb.

$$\text{The California Bearing Ratio} = \frac{20{,}000}{1000} = 20$$

Referring to the design curves for rigid surfacings in Fig. 85 from the left-hand diagram the concrete pavement thickness for a secondary road and California Bearing Ratio of 20 is ascertained to be 6 ins.

Therefore, the concrete-road section should consist of a rigid concrete road slab 6 ins. thick overlying a 3 in.-base layer of sand or clinker well rolled.

Note on Frost Action on Soils.

The effect of frost action on roads is twofold :—

(*a*) " Frost Heave," which is the lifting of the entire road surface, and, as such movement is not uniform, severe cracking of the pavement results.

(*b*) " Frost Boil," which occurs often in the British Isles during or after a thaw, and is due to softening of the sub-grade coupled with the effect of heavy traffic. The latter may cause such severe deformation of the pavement as to ruin the road, and an instance occurred early in 1940 on a long section of the Great North Road.

When a soil is susceptible to frost action it is frozen from the top downwards, and a series of ice lenses is formed in which the amount of heave increases with increasing moisture content. When the soil thaws, more water will be liberated than was originally present and, furthermore, the entire particle structure of the soil will have been loosened. With cohesive soils water is rapidly absorbed and a softening process occurs.

Frost action on roads is confined chiefly to coarse-grained cohesive soils, as the pores are large enough for water to rise easily and as such soils have a low plasticity index, they soften more readily than true clays. Frost action occurs with silts, silty clays, brick-earth soils of the Thames Valley and chalk, but such action is rarely serious with clays or gravelly soils. There is no sharp dividing line between susceptible and resistant soils, but approximately soils susceptible to frost action are those having 10 per cent. finer than 0·02 mm.

Sand Piles or Vertical Sand Drains

Vertical sand drains have been used in America for the stabilisation of soft compressible soils when the foundation soil is either too weak to support a proposed fill or structure, or is so compressible that large and continued settlements would occur after construction. The first application of this method was the stabilising of the mud foundation beneath the easterly roadway approach to the San Francisco–Oakland Bay Bridge. Laboratory and field experiments were made and results described by Mr. O. J. Porter in a Paper published in the *First International Conference of Soil Mechanics and Foundation Engineering* in 1936. Since then sand drains have been used for the stabilisation of weak and compressible soils beneath earth fills for many roadways and airfield fills. At first the use of sand drains were based upon an

empirical approach, but theoretical design methods are now being developed.

The design of sand drains depends upon the determination of the rate of consolidation of soft and compressible soils in which sand drains have been installed and subsequently loaded with the weight of fill material. Further vital matters in design are the shear strength of the soft soil, rate of gain of shear strength with consolidation, and the overall stability of the fill during its tipping.

The factors to be determined initially are:—

(a) The rate of tipping of the fill material.

(b) The amount and duration of surcharge fill loading.

(c) Amounts of settlement to be anticipated during and subsequent to the construction period.

(d) Pore water pressures to be used for the control of construction operations.

(e) Thickness of drainage blanket.

(f) Diameter and spacing of sand drains.

Basic work in the design of sand drains is due to Terzaghi, but in his theories two uncertainties proved to be difficulties which have only recently been overcome:—

(1) The effect of disturbance of the soil caused by the installation of the drains on the coefficient of consolidation. This disturbance includes a smearing action of the soil at the surface of the drain plus a disturbance of the soil for some distance from the drain, both of which affect the coefficient of consolidation.

(2) The effect of more rapid consolidation and hence settlement of soil near the sand drain in causing arching of the overlying soil and fill material.

Barron and Richart investigated the effect of the smeared zone and produced charts which indicate that the effect of smear is to reduce the effective diameter of the sand drain, and this is still a matter of judgment based on consolidation tests on undisturbed and remoulded samples.

The practical significance of sand drains is that they will probably not be effective in eliminating future settlements because they are only effective in accelerating primary consolidation; on the other hand they may be effective in increasing the rate of gain of shear strength and providing stability under loads otherwise not possible in very soft soils with large initial consolidation.

In studying the records of sand drain installations which have proved effective, the following data is extracted:—

Diameter of Sand Drain .	Range 6 ins. to 30 ins.	75% are between 18 ins. and 20 ins.
Drain spacing . . .	Range 6 ft. to 20 ft.	25% are 6 ft. to 8 ft. 73% are 6 ft. to 10 ft.

Difficulties which were experienced when constructing sand drains were (*a*) shear slides during construction, (*b*) slow rate of consolidation and (*c*) excessive settlement.

(*a*) In certain cases shear slides sheared the sand drains completely, causing them to become ineffective, and new sand drains had to be built. Normal stability analysis of shear slides should prevent this type of trouble.

(*b*) In several places consolidation took place so slowly that the tipping of fill had to be decreased and the surcharge could not be maintained for the length of time anticipated. In such cases the sand drains had been spaced too widely apart.

(*c*) Excessive settlement after construction. Certain roadway works had such rapid settlements that repaving was necessary shortly after construction had been completed. This difficulty was usually brought about by the surcharged loading period being too short, so that only a part of the initial consolidation took place. Tests on undisturbed samples should give results which will avoid this failure.

A " Review of the Uses of Vertical Sand Drains " is given by Messrs. P. C. Rutledge and S. J. Johnson in the *Highway Research Board Bulletin* No. 173, published by the National Academy of Sciences, 1958, Publication No. 533, Washington, D.C.

CHAPTER XV

SITE EXPLORATION AND SOIL INVESTIGATIONS

In this final chapter it is proposed to refer to methods of site exploration and soil sampling, as this volume would be incomplete without reference to this practical feature of soil mechanics, which deals with the interpretation of data obtained in the field. This is obviously an important branch of soil-mechanics studies, as the whole design of a structure should be based on field investigations, and inadequate or misleading information is useless.

A civil-engineering structure, however carefully designed, is no better than its foundations, and insufficient or inadequate information respecting the character and supporting capacity of the underlying soil may result in serious structural damage and distortion.

An engineer must always confine his conclusions to known facts, and must reason cause and effect, and approach his problems from a realistic viewpoint. Professional civil engineers have learnt to appreciate this, but some younger engineers are prone to depart from intelligent procedure when confronted with an intricate foundation problem, and fail to treat site conditions as they find them, but tend to consider them as they might wish them to be. Guesswork has no place in civil engineering, and must be eliminated.

Furthermore, an engineer must not allow his decisions to be affected in any way by claims of superiority and merit for various equipment and methods, but must determine by his own knowledge the efficacy of the measures he proposes to adopt. He must obtain sufficient and accurate data regarding the varied factors involved to be able to determine the correct procedure to adopt—one that will be safe, economical and effective.

The investigation of sub-soil conditions is one for which an engineer should be responsible, and if he feels his experience is insufficient to conduct the site exploration, then he should consult a specialist firm who are able to do so. Soil sampling has a two-fold importance : primarily it enables the engineer to design the foundations, and secondly it provides the contractor with accurate information so that procedure and construction methods may be planned. Furthermore, a contractor is unable to attribute difficulties to unexpected site conditions.

Foundation failures in general are due to a yielding of the

underlying soil, but if settlement is uniform over the area of the structure no serious damage is likely to occur. As referred to in Chapters IX, X, XI and XII, when unequal settlements take place secondary stresses are occasioned with the result that distortion occurs with attendant fractures due to re-adjustment of members. Such cracks may become so serious that the safety of the structure is involved and it is necessary to undertake remedial measures, which in some cases may be so expensive that reconstruction is more economical.

Site conditions may reveal that the expenditure necessary upon foundations may be so costly that they outweigh the advantages of location, transport facilities, purchase price and other considerations.

Site Exploration.

The amount of work involved to investigate satisfactorily the soil conditions at a site naturally depends on the importance of the works and what information may be obtained from other sources. Valuable data may be found from Geological Survey maps, records of previous bore-holes in the vicinity and foundation details of other structures erected in the neighbourhood. Sometimes it is only necessary to check up conditions which are known to exist.

Where little, if any, information is available, it is necessary to explore the site fully and the following three essentials must be obtained :—

(1) Type, sequence, thickness and dip of the strata.

(2) Undisturbed and disturbed soil samples for laboratory tests and identification respectively.

(3) Ground-water conditions.

It is necessary for sufficient borings and samples to be taken to include all the soil strata likely to have a bearing on the problem, and in this respect sufficient borings are required to indicate the variation in these conditions over the whole site. Undisturbed samples should be taken with each change of soil strata, and samples for identification purposes obtained every 3 ft.

Important features to which special attention should be paid are as follows :—

(1) The depths of the borings must include all stressed zones involved in the construction of the engineering works. They should be taken to a depth of at least one-and-a-half times the greatest width of the structure as mentioned in Chapter IX.

(2) A sufficient number of borings must be taken. Each site and problem varies so much that definite rules on the extent of exploration are difficult to lay down, and each instance must be considered on its merits. It may be that during the early part of the site investigations the need for more detailed or extensive exploration becomes evident, and the programme should be adaptable to allow for this.

(3) All water-bearing seams should be located. It is essential to ascertain the relative positions of layers of water-bearing sands or seams of silt located in beds of clay. The head of water pressure should be ascertained, and in this connection the Papers on the construction of the George V Graving Dock, published in the *Proceedings of the Institution of Civil Engineers*, are interesting in respect of artesian pressure.

(During the construction of this Dock it was necessary to excavate to a depth of 90 ft. below the dock coping in stiff, sandy clay which was overlying the sandy Bracklesham Beds. This sandy stratum existed only 10 ft. below the proposed excavation for the dock invert which would result in an uplift pressure of $2\frac{1}{4}$ tons per sq. ft. in open excavations due to the artesian head in the sand. Filter wells pumping over 700 gallons per minute were necessary to reduce this head.)

(4) Tidal variations in ground-water level must be ascertained. Important variations in ground-water pressure due to flood conditions or tidal levels may involve heads of pressure behind dock walls or other similar structures.

(5) Soft layers of clay, etc., may exist in alluvium. Under beds of compact sand, soft layers of clay may occur, which can cause the failure of a structure unless the strata is sufficiently explored and design made accordingly.

(6) Soft alluvium often has a hard crust which may be up to 10 ft. thick, and borings should be sufficiently deep to discover this. Holywell reservoir had a minor failure due to this feature.

(7) Deep beds of alluvium occur at river-mouths, which often have sudden changes in thickness. The depth of such layers may be considerable, and may exceed 100 ft., as at Saltash bridge, where the thickness is 150 ft., and at Gosport, where the thickness is 180 ft. Buried river-channels, may be filled with alluvium, which may reach considerable depths, and borings should be sufficiently deep to obtain requisite information.

(8) Large boulders must not be mistaken for bedrock. The failure of one large reservoir was attributed to a mistake

of this type, and usually reference to geological maps or a consultation with a geologist will avoid such errors. Geophysics enable rock to be ascertained within a margin of 5 ft., and either the "seismic refraction" or "electrical resistivity" method may be employed.

(9) The type of clay strata should be thoroughly investigated. It is important to determine whether a stiff clay is of the fissured type, such as the London, Gault, Weald and Kimmeridge clays, in order to interpret their likely behaviour under stress.

Soil Exploration Methods.

1. *Test Pits.* The most satisfactory method for the disclosure of soil-strata conditions is the sinking of a trial pit. The various strata can be examined in their natural state and ground-water conditions exactly ascertained. Undisturbed samples can be readily obtained.

The cost of test-pits increases rapidly with depth, and unless soil conditions are suitable the expense becomes uneconomical beyond a depth of 12 ft. Their depth is limited in certain soils by the nature of the material and difficulty of water control. Bore-holes are practically impossible in soils containing a large proportion of stones greater than 6 ins., and trial pits then become a necessity.

A record of the soils encountered must be made at the actual time of excavation of the trial pit, as the condition of the soils changes on exposure to air or on contact with water.

2. *Penetration Tests with Boring-Rods.* For the purpose of making a preliminary investigation of any site, penetration tests with a probing-rod, such as the "Mackintosh one-man bore-hole" outfit, is exceedingly useful. It can be very quickly assembled and exploration to a depth of 50 ft. performed in 45 minutes. The sinking of a 4-in. or 6-in.-diameter bore-hole is expensive, and often the sites chosen are found to be unsuitable owing to presence of large boulders or some other obstruction, thus preventing the driving of the bore-hole to its full depth. The probing rod indicates where bore-holes may be taken down to their full depth, and gives information leading to the siting of bore-holes at critical points.

A diagram of the "Mackintosh one-man bore-hole" equipment is given in Fig. 86. The apparatus consists of a number of steel rods of $\frac{1}{2}$-in. diameter, and each 4 ft. long. The rods are turned at the ends to take a full $\frac{1}{2}$-in.-diameter screw-thread, and they are connected up with barrel-shaped nipples of 1 in. diameter and 2 ins. long, with tapered ends. The spear-shaped driving point is 5 ins. long and $1\frac{1}{8}$-in.-diameter, and is driven with a cylindrical

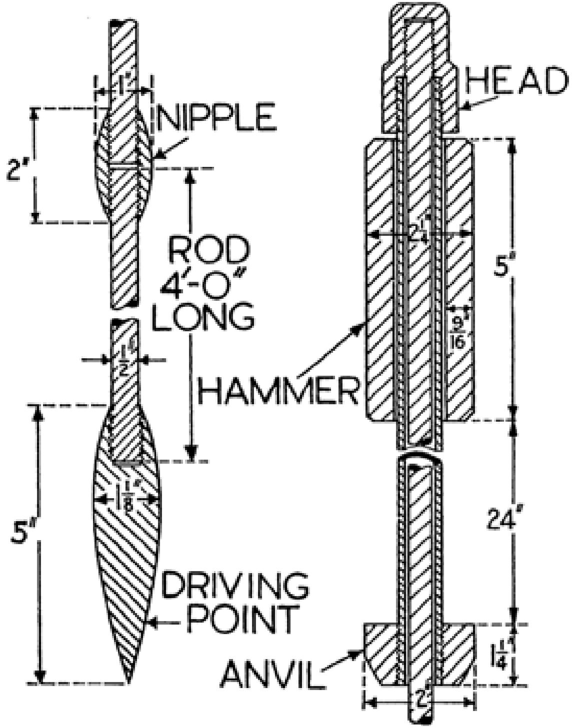


FIG. 86.—Mackintosh One-man Bore-hole Outfit.

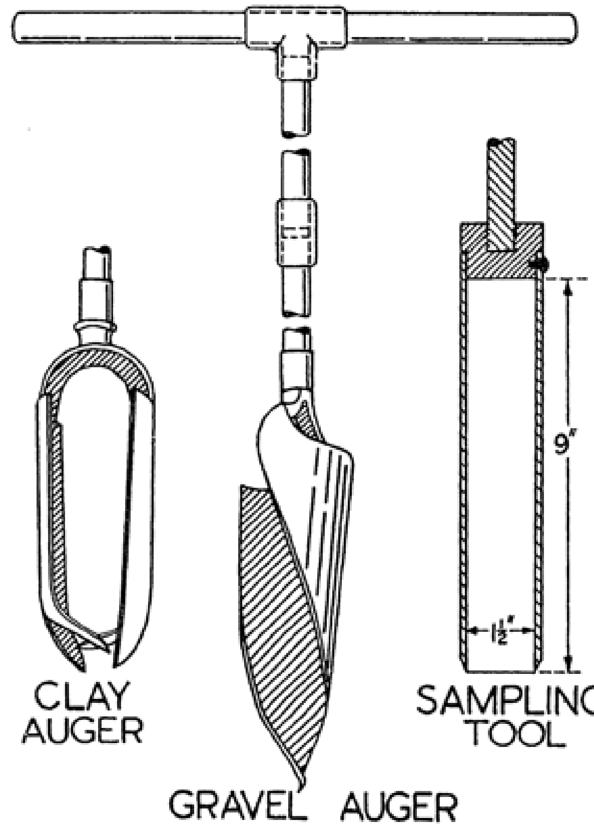


FIG. 87.—Post-hole Auger.

solid steel block 5 ins. long and $\frac{9}{16}$ in. thick, which slides freely on a rod. The hammer delivers a blow in falling a distance of 2 ft., and the number of blows required to drive 1 ft. of the rod is periodically recorded. New sections of rod are added as the point is driven into the soil.

Samples for inspection are taken by a sampling tube 2 ft. long and $\frac{3}{4}$-in. external diameter, with $\frac{3}{8}$-in. bore. Various bits are used for rocks, and augers for clays or sands.

3. *Post-Hole Auger*. In clays, satisfactory unlined bore-holes may be sunk to a depth of 30 ft. with a post-hole auger, which is illustrated in Fig. 87. The equipment consists of an auger suitable for clay, which is screwed to a length of gas-piping, to which further lengths may be added as the bore-hole is sunk. The auger is turned by means of a tee-piece handle made of gas-tubing.

When undisturbed samples are required for compression tests in the Building Research Station apparatus, the auger and rods are taken from the bore-hole, and a sampling tube attached in lieu. The sampler as shown in Fig. 87 is 9 ins. long and $1\frac{1}{2}$ ins. in diameter, which is driven into the clay, given a half twist and withdrawn. Samplers must be of dimensions which conform to the following :—

$$\text{Area ratio} = \frac{\text{Area of steel in cutting edges}}{\text{Area of sample}} < 25\%.$$

For gravels a special type worm auger is used, but with such soils and sands undisturbed samples cannot be obtained.

4. *Hand-boring Tackle with Winch*. For the purpose of sinking trial holes to a depth of 60 ft., or even up to 100 ft., it is necessary to use a hand-boring tackle, which should preferably be fitted with a winch. Photographs in Figs. 88 and 89 show the apparatus in use. The equipment is set up so that the head-pulley supported by the sheerlegs is plumbed over the centre of the proposed bore-hole.

In clays the borings are made with a clay auger without lining tubes. In compact beds of sand or gravel the soil is broken up by a chisel and a worm-auger. The core is removed by means of a shell with a valve which retains the loose particles. Lining tubes to the bore-hole are not usually provided with compact sand or gravel beds, but it is essential to sink tubes with loose sand or gravel.

For sinking tubes a cutting-shoe is screwed to the lower end of the first tube and a driving-cap to the upper end. The rope is taken three times round the barrel of the winch, whilst the free end is threaded over the head-pulley and attached to a spring hook. The monkey with leader is hung on the springhook and

hoisted until the lower end of the leader can be inserted in the driving-cap of the tube to be sunk.

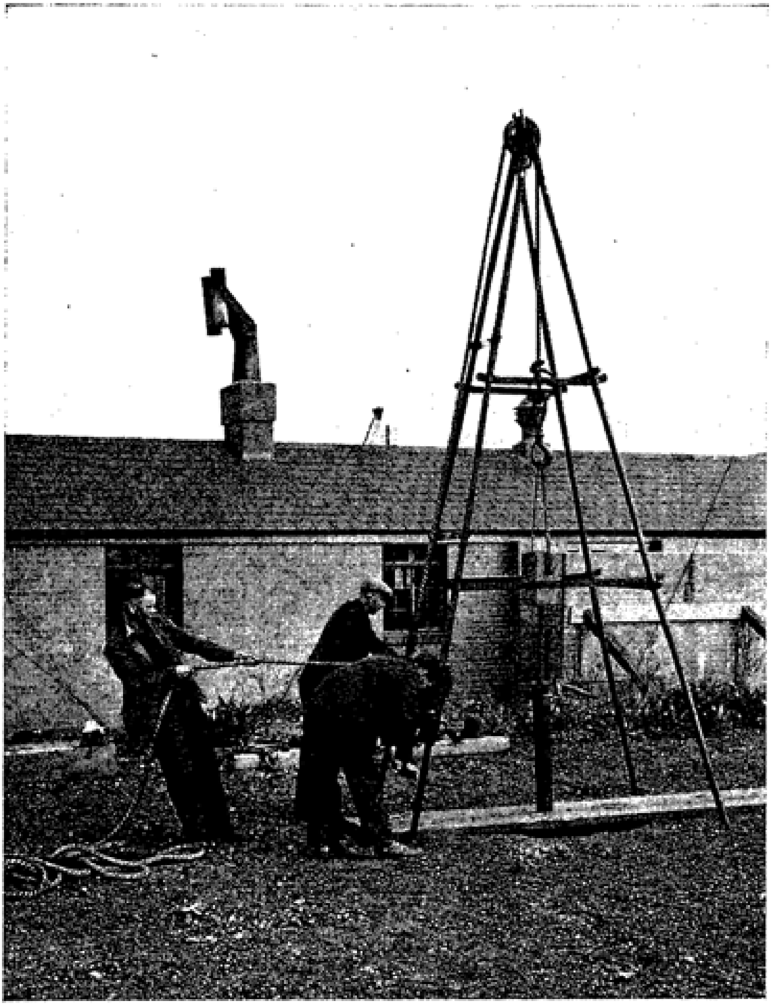


FIG. 88.—Hand-boring Tackle Erected and Indicating Method of Driving the Tube by Means of the Monkey and Leader.

One man holds taut the loose end of the rope, while the other two men turn the winch-handles, thus raising the monkey and leader a distance of 24 ins. The operator releases the loose end of the rope, thus allowing the monkey to fall and deliver a blow

upon the driving-cap. This operation is repeated until the tube has been sunk to a sufficient depth, and the core is then removed

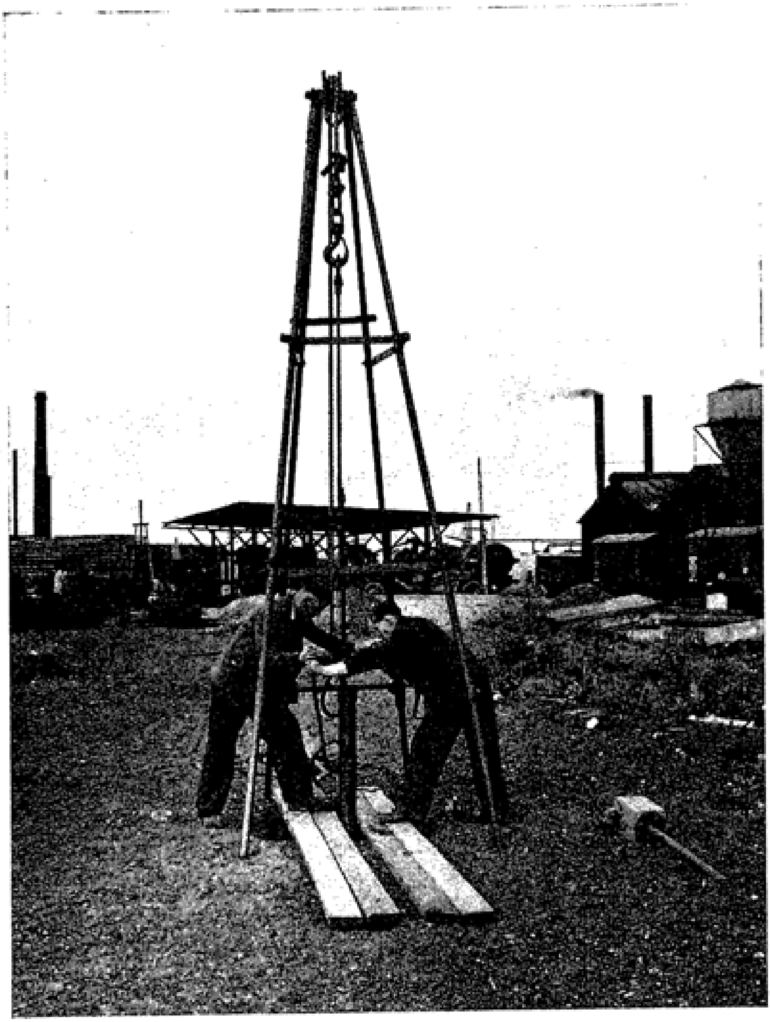


Fig. 89.—Hand Rotating the Boring Tool by Means of Rod Tillers Attached to the Boring-rods.

by auger or shell. The photograph in Fig. 88 shows the method of driving the tube, and in Fig. 89 the method is shown for rotating the boring-tool by means of rod-tillers attached to the boring-

rods. Boring-rods accidentally dropped down the bore-hole are recovered by means of a tool known as a "crowsfoot."

The tubes may be removed by means of the hand-winch or by screw-jacks. Soil sampling may be made with the sampler previously described or by larger samplers of the type indicated in Fig. 90. The cutting edge of this tool is slightly smaller in

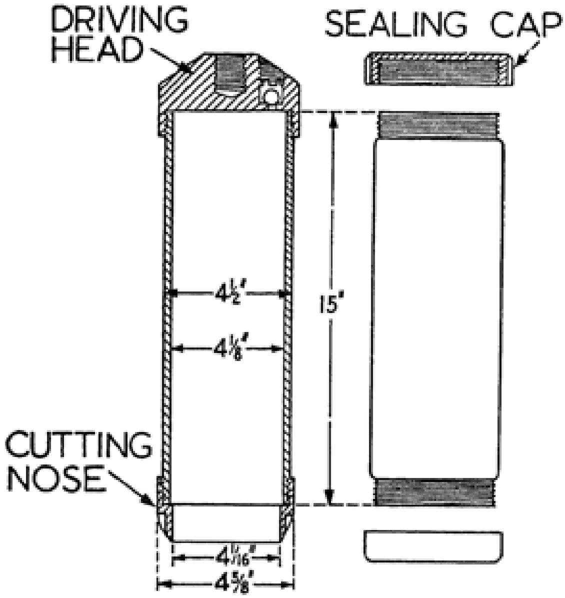


Fig. 90.—Sampling Tube.

diameter than the interior of the cylinder. The shell is $\frac{3}{16}$ in. thick, as it is essential to keep the cross-sectional area of the metal to a minimum to obtain a small "area ratio." The adaptor head is tapped to take boring-rods and is fitted with a non-return valve. The cutting nose and head can be removed and end-caps fitted in place, which protects the sample during transport to the laboratory and prevents drying out.

At the laboratory, measurement is made of the natural water content of all samples and liquid limit, and other index tests made on a selected number of samples. From such tests soil classification and characteristics can be determined in the manner described in the first two chapters.

The Cox Bolt Gun.

Undisturbed samples of friable materials, such as weakly

TABLE 21.

Classification of Soils.

TYPE OF SOIL.	SIZE OF PARTICLES.	STRENGTH.	FIELD IDENTIFICATION.	STRUCTURE.	SIMILAR MIXED TYPES.	REMARKS
COHESIONLESS SOILS.						
1. Gravels.	Particles exceed B.S. sieve No. 7.	(a) *Compact.* Picks used for excavation. (b) *Loose.* Spades used for excavation.	Particles greater than $2\frac{1}{4}$ in. diameter may be classified as boulders.	Stratified or homogeneous.	Sandy gravels. Boulder gravels. Clayey gravels (Hoggin).	
2. Sands.	(a) *Coarse sand.* Particles pass B.S. sieve No. 7 but retained on No. 25. (b) *Medium sand.* Particles pass B.S. sieve No. 25 but retained on No. 72. (c) *Fine sand.* Particles pass B.S. sieve No. 72 but retained on No. 200.	(a) *Compact.* Picks used for excavation. (b) *Loose.* Spades used for excavation.	Particles visible to naked eye. No cohesion when dry. In addition to the above "fine sands" exhibit dilatancy. (See note *.)	Stratified or homogeneous.	Well-graded sands. Silty sands. Clayey sands. Shelly sands.	*Dirty sands.* Rub a sample in the hands, if (a) a little soiled consider a "good sand." (b) very dirty treat as a "silt." *Quicksand.* If an unbalanced hydrostatic head produces an upward flow of water through the soil the grains of sand may go into suspension creating a condition known as a "quicksand."
3. Silts. (Inorganic.)	Most particles pass a B.S. sieve No. 200 but have a greater diameter than 0·002 mm.	(a) *Firm.* (b) *Soft.*	Particles invisible or barely visible to the naked eye. Gritty touch. Exhibit dilatancy. (See note *.) Exhibit very slight cohesion when dry. No plasticity.	Stratified or homogeneous.	Clayey silt. Sandy silt. Micaceous silt.	Silts are sometimes misclassified as clays but as all types of silts are treacherous soils they should be definitely identified in the manner shown. Silts are often highly compressible and if a disturbance in ground water conditions should occur their structure may change and serious settlement take place.
Organic Silt.	Ranges from light to dark grey and may be found mixed with sands, shells, vegetable matter or peat. With a high water content it is often referred to as mud or river bed ooze. A sample rolled into a ball and exposed to atmospheric conditions for a few days will show, when broken open, annular rings of discoloration due to oxidation of vegetable matter. It is highly compressible and comparable with peat, but if an appreciable sand content is present, then it may be capable of supporting light loads.					

COHESIVE SOILS.						
Clays.	Consists of particles smaller than 0·002 mm. diameter.	(*a*) *Stiff.* Cannot be moulded by the fingers. Picks used for excavation. (*b*) *Medium.* Can be moulded by strong pressure by the fingers. Spades used for excavation. (*c*) *Soft.* Easily moulded by fingers. Shovels used for excavation.	Smooth touch. Plasticity. No dilatancy. Considerable cohesion when dry.	Fissured, laminated, intact or weathered.	Boulder clay. Sandy clay. Silty clay. Marls. Organic clay.	
ORGANIC SOILS.						
Peats.	Fibrous material.	(*a*) *Firm.* (*b*) *Soft.* Very compressible and spongy.	Identified visually. High compressibility. Fibrous texture. Brown or black in colour.	Fibrous.	Sandy peat. Silty peat. Clayey peat.	Peat is partially carbonised vegetable matter; it is composed of annual deposits of dead vegetation protected from decomposition by continuous submergence. Will not support any additional constantly applied load without considerable reduction in volume.

Other types which may occur are: Top soil and loam (usually indicate original ground level). Soft chalk. Earthy chalk. Soft coal. Coal. Ballast. Ashes.

* *Dilatancy.*—Dilatancy is a property of a soil which may be tested in the following manner: If a sample in the wet state is shaken horizontally in the hand, the surface becomes wet and shiny. If the sample is then compressed between the fingers, the water disappears and the surface becomes dry and dull. Shaking causes the water to appear again.

N°..........

BORING RECORD.

SITE.......................... NEAREST STATION. ....

N° OF BOREHOLE.............. SCHEME FOR WHICH BOREHOLES OBTAINED} ....

DATE........WEATHER........... N° OF MEN EMPLOYED.................

TIME DETAIL.

Hours Worked	Men Hours Worked	Time Lost	Reason	Total Time

Setting up Tackle	Dismantling	Transporting to New Site or Loading up
Hours	Hours	Hours

DEPTH BORED FROM...............TO......................BELOW GROUND LEVEL.

LEVELS:— R L in vicinity of Borehole ....Feet O.D.................

Ground Level of Borehole ....Feet O.D.................

Peg Level (For T B M)...................Feet O.D.................

WATER LEVELS:— At Beginning of Work ....

At End of Work ....

If Artesian pressure of water encountered, state level when found ....

TOTAL PENETRATION.............................Length of Tube driven ....

Length of Tube Jacked up } ....

BOREHOLE REPORT DETAILS.

Levels Referred to O.D			Description of Soil (See List)	N° of Blows for Driving Tube 6 in ground	If Chisel used for Breaking up Hard Material	General Colour of Layer	Degree of Moisture above Water Level	Sample N°
Top of Layer	Bottom of Layer	Thickness of Layer						

SIGNATURE.......................

Date of Report......................

FIG. 91.—Typical Boring Record.

cemented sands or weathered chalks, marls, shales, are difficult to obtain by the foregoing methods. Attempts which have been made to sample these materials using tube samplers result only in driving the sampler a few inches, and this makes it difficult to recover the sample, and even if it is recovered it is likely to be too short for testing in compression. Although on occasions, it may be possible to obtain full penetration of a tube sampler into a weakly cemented soil, the action of driving the tube is liable to break up the sample into a mass of crumbly fragments. Coring by diamond drill is possible in strongly cemented sands, but with weakly cemented material the drilling water is liable to flush away the core to such an extent that it cannot be retained in the core-barrel, and extraction is impossible.

In order to overcome these difficulties a Cox submarine bolt gun has been successfully adapted as a sampling gun. This gun consists of an outer barrel sliding on an inner barrel. An explosive charge is placed at the upper end of this inner barrel. At the base of this charge is a plug provided with a waist of such dimensions that it fails in tension as soon as the exploding charge has built up sufficient pressure. The plug is attached to the bolt, and the energy released when the plug breaks, drives the bolt through the inner barrel with such velocity as to penetrate a steel plate. The gun is adapted to fire a sampling tube. The gun is lowered into the borehole casing, the safety catch released, and the whole unit lowered by the drill rope until slackness indicates the bottom of the borehole has been reached. The slack is taken up and then released, allowing the outer barrel to fall and fire the charge. The gun and sample are raised together. The charge weighed can be varied to obtain the required penetration in materials of varying hardness.

This method enables core samples of soft rocks to be obtained without rotary drilling, and the need for bringing in a diamond drill to prove for a short distance is eliminated. The adaptation and development of the Cox Bolt gun has been carried out by members of the Central Laboratory Staff of George Wimpey & Co., Ltd.

Boring Records

It is essential for proper and adequate boring records to be made during the sinking of a bore-hole. Borings are expensive, and the utmost information of any value should be obtained from them. Fig. 91 sets out a typical boring record.

The description of the soil should be in accordance with Table 21, and should give the general colour of the soil, the degree of dampness, if above water level, and if traces of peat or vegetable

matter are present. The data obtained from this record enable the cost of the bore-holes to be determined, as well as giving the necessary information relating to the soil strata themselves. The cores from which samples are not taken should be laid out for inspection near the site in the sequence in which they are extracted.

Field Identification of Soils

The exact character of a soil can be defined in many cases only by mechnical analysis or examination in the laboratory, but it is essential for any soil to be classified in the field and described in terms that are generally accepted and understood.

Generally, it is sufficient to recognise five types: Gravels, Sands, Silts, Clays, Peat. A further sub-division of gravels, sands and silts can be conveniently made by the three grades, coarse, medium and fine, according to particle size.

Table 21 sets out a general classification of soils with field identification tests.

Problem 96. If you were arranging for site exploration with bore-holes in connection with the proposed construction of a two-storey factory building about 40 ft. square in plan, and adjacent to the bank of a tidal river, what bore-holes would you propose to sink? Indicate depths, numbers and types of samples to be taken. If a sampler of 2 ins. internal diameter is to be made for extracting undisturbed samples, what maximum thickness of shell would you recommend?

The bore-holes should be spaced at distances of 75 ft. along the river-bank in rows of three, spaced 50 ft. apart, with the first row 30 ft. from the river-bank. According to variations in the strata, additional bore-holes may be necessary.

The depth of the bore-holes should be at least one-and-a-half times the greatest width of the structure, and therefore the bore-holes should be taken down to 60 ft. If the top stratum is alluvium, it is necessary to sink bore-holes unitl a hard stratum is reached, which may be a greater depth than 60 ft.

Undisturbed samples should be taken with every change of strata, and in any case at each 6 ft. depth of bore-hole sunk. Samples for identification purposes should be taken at every 3 ft. depth, and the remainder of the cores laid out for inspection in their correct order.

The sampler tube should have an area ratio less than 25 per cent.

$$\frac{\text{Area of steel in cutting edge}}{\text{Area of sample}} = 0{\cdot}25 \text{ (max.)}$$

Hence, thickness of shell

$$=\sqrt{\frac{0{\cdot}786}{\pi}+1}-1=\frac{7}{64}\text{ in. (max.)}$$

A tube well should be sunk in one of the bore-holes on the river-bank for recording tidal changes in ground-water level. This

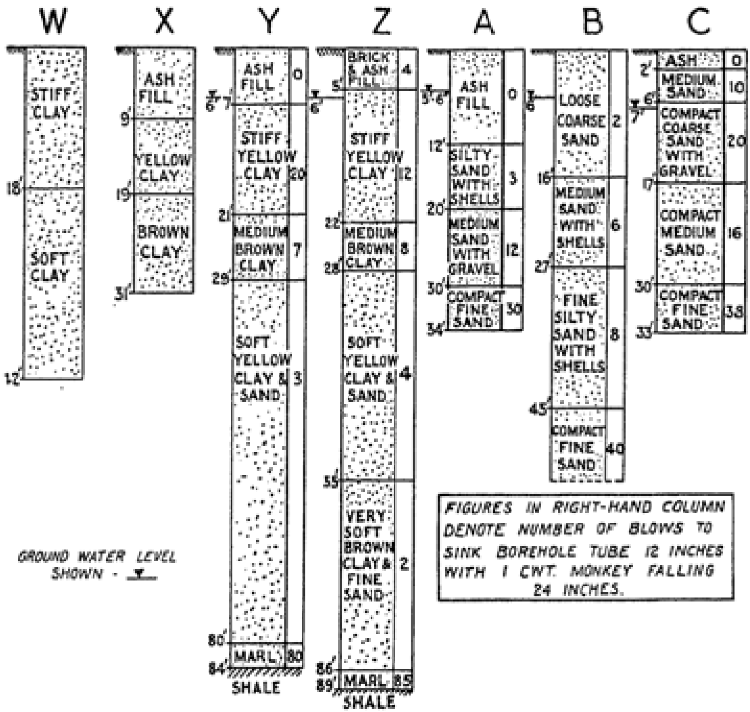


Fig. 92.—Bore-hole Data. (Problems 97 and 98.)

may consist of a 12-ft. length of standard 2-in.-diameter galvanised iron tube, the lower end of which is perforated for a length of 2 ft. 6 ins. If necessary, the tube length may be greater to suit site conditions. The pipe is inserted in the bore-hole, and the space between the pipe and bore-hole carefully packed with $\frac{3}{8}$-in. chippings for the lower 3 ft. The remainder of the pipe is packed with such porous filling as may be available—gravel, ashes or chippings—to within 3 ft. of ground level. An earthenware drain-pipe can be inserted at ground level to retain the sides of the bore-hole, and this upper 3 ft. of tube is tightly packed with clay to prevent surface water entering. A cap to the pipe should be

provided, and the variations in the level of the water due to tides or other conditions may be checked by means of a dip-rod inserted in the tube well.

Problem 97. The bore-holes " W " and " X " as shown in Fig. 92 are typical of those which were made for a building 50 ft. long and 30 ft. wide. When the building was erected the front wall moved horizontally 4 ins. and settled 4 ins. What are your observations on these borings?

First, the bore-holes were not deep enough. For a structure 50 ft. long they should have been taken down to a depth of at least 75 ft.

Secondly, the borings were stopped in soft clay strata, which is most unsatisfactory. Bore-holes should always be taken down to a hard stratum, and the information contained in " W " and " X " is useless.

Thirdly, the information conveyed by the borings is meagre, and gives no indication regarding the bearing capacity of the soils through which the bore-hole passes.

After the subsidence of the building, borings of the type " Y " and " Z," Fig. 92, were made, and these are the borings which should have been made first of all. In this instance it was necessary to drive piles and carry the walls on needle-beams. At the time when this work was carried out, the cost of piling and remedial works amounted to £4,000, but had piles been provided in the first instance they would have cost £900.

Problem 98. In connection with the construction of a new arterial road passing under a railway track, bore-holes " A," " B " and " C," as shown in Fig. 92, were sunk to ascertain soil conditions most suitable for siting a bridge-pier. What observations can be made upon these bore-hole results?

The bore-hole " A " indicates that suitable strata does not exist until a depth of 20 ft. is reached, where medium sand with gravel exists, the upper layers consisting of fill material and silty sand. Ground-water level is 5 ft. 6 ins. below ground level, and pumping of the excavations would be necessary.

Conditions in bore-hole " B " are worse, and a suitable strata cannot be found until a depth of 45 ft. is reached.

The site of bore-hole " C " provides the best foundation as little filling material has been deposited, and excavations need be taken to 6 ft. only, where compact coarse sand and gravel exist. Ground-water level is 7 ft. below ground level, and the necessity of pumping excavations would be avoided.

In this instance foundations at site " A " would have cost £9,500, whereas the actual cost at site " C " was £5,150. The original lay-out of the new road was slightly altered to take advantage of the site conditions at " C."

SELECTED BIBLIOGRAPHY ON SOIL MECHANICS

AHMED, S. B. "A Theory of Earth Pressure as applied to Retaining Walls." *J. Instn. Civ. Engrs.*, Vol. 9, Oct. 1938.

ALBERT, S. G. and A. CASAGRANDE. "Research on the Shearing Resistance of Soils." Institute of Technology, Mass., U.S.A., 1932.

ALLEN, H., and C. A. HOGENTOGLER. "Important Considerations in Soil Mechanics." *Bulletin A.S.T.M.*, Oct. 1938.

ALLIN and NACHSLEN. "Power-house Foundations and Circulating Water Tunnels at the Ford Motor Co's. Works, Dagenham." *Proc. Instn. Civ. Engrs.*, Vol. 235, 1933.

BELL, A. L. "The Lateral Pressure and Resistance of Clay and the Supporting Power of Clay Foundations." *Proc. Instn. Civ. Engrs.*, Vol. 199, 1914.

BELL, A. L. "Pressure and Resistance of Clays." *Proc. Instn. Civ. Engrs.*, Vol. 199, 1914–1915, Part 1.

BEVAN, R. C., and C. F. JENKIN. "Earth-Pressure Tables." H.M.S.O., London, 1934.

BINNIE, G. M., and J. A. PRICE. "An Apparatus for Measuring the Lateral Pressure of Clay Samples under a Vertical Load." *J. Instn. Civ. Engrs.*, Vol. 15, Feb. 1941.

BOUSSINESQ, J. "Application des Potentiels a l'étude de l'équilibre et du mouvement des solides elastiques." Gauthier-Villars, Paris, 1885.

BUCHHOLTZ, W. "Erdwiderstand auf Ankerplatten." Thesis at the Technische Hochschule Hannover, 1931–1933.

CASAGRANDE, A. "Characteristics of Cohesionless Soils affecting the Stability of Slopes of Earth Fills." *J. Boston Society of Civ. Engrs.*, Harvard, U.S.A., Vol. 23, Jan. 1936.

CASAGRANDE, A. "The Structure of Clay and its Importance in Foundation Engineering." Mass. Inst. of Technology, 1932.

COOLING, L. F. "Soil Mechanics and Site Exploration." *J. Instn. Civ. Engrs.*, Vol. 18, Mar. 1942.

COOLING, L. F., and H. Q. GOLDER. "Portable Apparatus for Compression Tests on Clay Soils." *Engineering*, Jan. 19th, 1940.

COOLING, L. F., and H. Q. GOLDER. "The Analysis of the Failure of an Earth Dam during Construction." *J. Instn. Civ. Engrs.*, Vol. 19, Nov. 1942.

COOLING, L. F., and A. W. SKEMPTON. "Some Experiments on the Consolidation of Clay." *J. Instn. Civ. Engrs.*, Vol. 16, June 1941.

COOLING, L. F., and D. B. SMITH. "Shearing Resistance of Clay Soils." *J. Instn. Civ. Engrs.*, Vol. 3, 1936.

COULOMB, C. A. "Essai sur une Application des Regles des Maximis et Minimis a quelques Problemes de Statique." *Royale Academie des Sciences*, Paris, 1776, Vol. 7.

CROSTHWAITE, P. C. "Experiments on Earth Pressure." *Proc. Instn. Civ. Engrs.*, Vol. 103, 1916.

DAVIES and JACOBY. "Foundations of Bridges and Buildings." McGraw Hill, London, 1941.

DORE, J. M., and F. L. PLUMMER. "Soil Mechanics and Foundations." Pitman, New York, 1940.

FOX, C. S. "Civil Engineering Geology." Technical Press, London.

GOLDER, H. Q. "The Ultimate Bearing Pressure of Rectangular Footings." *J. Instn. Civ. Engrs.*, Vol. 17, Dec. 1941.

GOLDER, H. Q. "An Apparatus for Measuring the Shear Strength of Soils." *Engineering*, Vol. 153, June 26th, 1942.

GORSBIE, COL. R. E. "Geology for Engineers." Griffin.

HOGENTOGLER, C. A. "Engineering Properties of Soil." McGraw Hill, London, 1937.

JENKIN, C. F. "The Pressure on Retaining Walls." *Proc. Instn. Civ. Engrs.*, Vol. 234, 1932.

KREY, H. "Erddruck, Erdwiderstand and Tragfahigkeit des Baugrundes." W. Ernst & Sohn, Berlin, 1936.

KRYNINE, D. P. "Soil Mechanics." McGraw Hill, New York, 1941.

MARKWICK, A. H. D. "Road Construction on Peat Foundations." *Roads and Road Construction*, Vol. 17, 203, 1939.

MARKWICK, A. H. D. "The Basic Principles of Soil Compaction and their Application." Instn. Civ. Engrs., Road Paper No. 16, 1945.

MARKWICK, A. H. D. ' Soil Mechanics in Road and Aerodrome Construction." *J. Instn. Civ. Engrs.*, Vol. 18, March 1942.

MINIKIN, R. R. "Clay and Soil Mechanics." *Engineering*, Sept. 26th and Oct. 30th, 1941.

MINIKIN, R. R. "Stability of Clay Slopes. Analysis by the Swedish Method." *Civil Engineering*, Nov. 1940.

NIEBUHR. "Die Bautechnik." Vol. 17, 1939.

PIRUM, G. B. R. "Foundations." *Struct. Engr.*, Vol. 12, 1934.

REYNOLDS, H. R. "Soil Mechanics and the Railway Engineer." Instn. Civ. Engrs., Railway Paper No. 10, Dec. 1943.

SKEMPTON, A. W. "Settlement Analysis of Structures." *Engineering*, Sept. 30th, 1938.

SKEMPTON, A. W. "Investigation of the Bearing Capacity of Soft Clay Soil." *J. Instn. Civ. Engrs.*, Vol. 18, June 1942.

STROYER, J. P. R. N. "Earth Pressure on Flexible Walls." *J. Instn. Civ. Engrs.*, Vol. 1, Nov. 1935.

TAYLOR, PROF. D. W. "Stability of Earth Slopes." *J. Boston Soc. Civ. Engrs.* Vol. 24, July 1937.

TERZAGHI, K. VON. "Soil Mechanics—A New Chapter in Engineering Science." *J. Instn. Civ. Engrs.*, Vol. 12, June 1939.

TERZAGHI, K. VON. "Theoretical Soil Mechanics." John Wiley & Sons, 1943.

TIMOSHENKO, S. "Theory of Elasticity." *Phil. Trans. Roy. Soc.*, 1934.

TSCHEBOTAREFF, G. "Clay Analyses and Forecasting Settlements." *Struct. Engr.*, Vol. 12, Aug. 1934.

WENTWORTH-SHIELDS, F. E. "Stability of Quay Walls on Earth Foundations." *J. Concrete Inst.*, Vol. 6, 1913.

WILSON, G. "The Calculation of the Bearing Capacity of Footings on Clay." *J. Instn. Civ. Engrs.*, Vol. 17, Nov. 1941.

WOLLASTON, C. HYDE. "Foundations and Earth Pressures." Hutchinson, London, 1939.

REPORTS, ETC.

American Society of Civil Engineers. "Practical Application of Soil Mechanics—A Symposium." *Proc. Am. Soc. Civ. Engrs.*, 1937.

Proceedings of the Purdue Conference on "Soil Mechanics and its Applications." Purdue University, Lafayette, Indiana, U.S.A., 1940.

"Soils, Concrete and Bituminous Materials." Precis of course given at the Road Research Laboratory, Harmondsworth. H.M.S.O., 1945.

Swedish Geotechnical Commission, 1922. "Statens Jarnvagars Geotekniska Kommission, 1914–1922," Stockholm. (Summary in English.)

INDEX

2.2.74